

M.Sc. Semester-3 Examination

504-EA

Mathematics

Time : 2-30 Hours]

March-2024

[Max. Marks : 70]

Q.1

Solve the following LPP using the Two-Phase Method

14

(i) $\text{Minimize } Z = x_1 - 2x_2 - 3x_3$ subject to constraints: $-2x_1 + x_2 + 3x_3 = 2$; $2x_1 + 3x_2 + 4x_3 = 1$; $x_1, x_2, x_3 \geq 0$ (ii) Use the post-optimal optimization method to find the range of cost coefficient (in objective function) of x_1 so that the current optimal solution remains optimal.For the LPP: $\text{Maximize } Z = 4x_1 + 6x_2 + 2x_3$ subject to the constraints: $x_1 + x_2 + x_3 \leq 3$; $x_1 + 4x_2 + 7x_3 \leq 9$; $x_1, x_2, x_3 \geq 0$

OR

(i) $\text{Maximize } Z = -3x_1 - 2x_2$

14

subject to the constraints: $x_1 + x_2 \geq 1$; $x_1 + x_2 \leq 7$; $x_1 + 2x_2 \geq 10$; $x_2 \leq 3$; $x_1, x_2 \geq 0$
using the dual simplex method.

(ii) Solve using the Branch & Bound method.

 $\text{Maximize } Z = 6x_1 + 8x_2$ subject to the constraints: $4x_1 + 16x_2 \leq 32$; $14x_1 + 4x_2 \leq 6$; $x_1, x_2 \geq 0$ are integers.

Q.2

(i) Find the critical path?

14

| ACTIVITY | PREDECESSOR | DURATION |
|----------|-------------|----------|
| A | --- | 4 |
| B | A | 4 |
| C | B | 4 |
| D | B | 9 |
| E | B | 16 |
| F | C | 2 |
| G | C | 1 |
| H | F,G | 2 |
| I | D,H | 4 |
| J | E,I | 2 |
| K | J | 2 |

Consider a project whose activities are given below:

(ii)

| Activity | 1-2 | 1-3 | 1-4 | 2-5 | 2-6 | 3-4 | 3-6 | 4-6 | 4-7 | 5-8 | 6-8 | 6-9 | 7-9 | 8-9 |
|----------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Time | 2 | 6 | 4 | 6 | 8 | 3 | 3 | 7 | 2 | 5 | 4 | 3 | 13 | 7 |

1. Draw the network

2. Calculate the total float.

OR

(i) Consider a project consisting of 11 jobs with the following precedence relations and estimates. 14

| Activity | Predecessor | Duration(weeks) | | |
|----------|-------------|------------------------|-------------------------|----------------------|
| | | Optimistic time (a) | Most likely time (m) | Pessimistic time (b) |
| A | - | 6 | 7 | 8 |
| B | - | 1 | 2 | 9 |
| C | - | 1 | 4 | 7 |
| D | A | 1 | 2 | 3 |
| E | A,B | 1 | 2 | 9 |
| F | C | 1 | 5 | 9 |
| G | C | 2 | 2 | 8 |
| H | E,F | 4 | 4 | 4 |
| I | E,F | 4 | 4 | 10 |
| J | D,H | 2 | 5 | 14 |
| K | I,G | 2 | 2 | 8 |

Find the probability for completing the project on or before 25 weeks?

(ii) The data on normal time and cost along with crashed time and cost associated with a project is:

| Activity | Immediate | Duration (Days) | | Cost (Rs.) | |
|----------|-----------|--------------------|-------|---------------|-------|
| | | Normal | Crash | Normal | Crash |
| A | -- | 10 | 7 | 20 | 30 |
| B | -- | 8 | 6 | 15 | 20 |
| C | B | 5 | 4 | 10 | 14 |
| D | B | 6 | 4 | 11 | 15 |
| E | B | 8 | 5 | 9 | 15 |
| F | E | 5 | 4 | 5 | 8 |
| G | A, D, C | 12 | 8 | 3 | 4 |

Using crash analysis crash activity (Maximum 2) to find minimum project cost with indirect cost Rs. 400 per day.

Q.3

(i) Find and classify the critical points of the function $f(x, y) = x^2 + 2xy + 2xz + 2y^2 + 4yz + 3z^2$. 14

(ii) Use Kuhn-Tucker Conditions to solve the following non-linear Programming Problem

$$\text{minimize } z = (x_1 + 1)^2 + (x_2 + 2)^2$$
Subject to conditions: $0 \leq x_1 \leq 2$ and $0 \leq x_2 \leq 1$.

OR

(i) Use Langragian multiplier method to solve $\text{Min } Z = 4x_1^2 + 2x_2^2 + x_3^2 - 4x_1x_2$ 14

Subject to constraints: $x_1 + x_2 + x_3 = 15$; $2x_1 - x_2 + 2x_3 = 20$; $x_1, x_2, x_3 \geq 0$

(ii) Solve Quadratic Programming Problem using Wolfe's Method

$$\text{maximize } z = 2x_1 + x_2 - x_1^2$$

Subject to constraints: $x_1 + 3x_2 \leq 6$, $2x_1 + x_2 \leq 4$; $x_1, x_2 \geq 0$

Q.4

(i) A vessel is to be loaded with stock of 3 items. Each unit of item I has a weight and value. The maximum cargo weights the vessel can take is 9 and the details of the three items are as follows: 14

| Item (i) | Weight (w_i) | value(r_i) |
|----------|------------------|----------------|
| 1 | 1 | 30 |
| 2 | 3 | 50 |
| 3 | 2 | 25 |

Develop the recursive equation for the above case and find the most valuable cargo load without exceeding the maximum cargo weight using dynamic programming.

(ii) Solve the following FLPP

$$\text{maximize } z = \frac{-3x_1 - x_2}{x_1 + 2x_2 + 5}$$

Subject to constraints: $x_1 + x_2 \geq 1$, $2x_1 + 3x_2 \geq 2$; $x_1, x_2 \geq 0$

OR

(i) Solve the following LFPP:

$$\text{Max } Z = \frac{x_1 + 3x_2 + 2x_3}{2x_1 + x_2 + 4x_3 + 1}$$

subject to constraints: $x_1 + 3x_2 + 6x_3 \leq 8$, $2x_1 + 2x_2 + 4x_3 \leq 5$ and $x_1, x_2, x_3 \geq 0$.

(ii) Use dynamic Programming to show that: $p_1 \log p_1 + p_2 \log p_2 + \dots + p_n \log p_n$

subject to constraint $p_1 + p_2 + \dots + p_n = 1$ and $p_i \geq 0$ for all i
is minimum when $p_1 = p_2 = \dots = p_n = \frac{1}{n}$.

Q.5 Attempt any **SEVEN**

14

1. What is the nature of the solution for primal problem if its dual has unbounded solution?

2. Why rounding off solution values of decision variable is not acceptable in integer linear programming problem?

3. Define: Redundant constraint

4. Mention at least two differences between PERT and CPM techniques.

5. Mention formula to find Crash cost slope for critical activities

6. Negative value of total float for non-critical activity indicates

- Project completion is behind the schedule date
- Project completion is ahead of the schedule date
- Project completion follows schedule date
- Incomplete information

7. Find the number of units of input require to obtain maximum value of output for the production function $Q = 40F + 3F^2 - \frac{F^3}{3}$, where Q is the total output and F is the units of input.

3

P.T.O

8. Check whether the matrix $\begin{bmatrix} 1 & 1 \\ 1 & -2 \end{bmatrix}$ is negative definite, positive definite or indefinite?
9. What is free variable and use of free variable in Beale's method.
10. How does dynamic programming conceptually differ from linear programming?
11. The Return function in Dynamic programming depends on
 - a) stages
 - b) states
 - c) alternatives
 - d) all the above
12. Number of stages and number of states for the given LPP

$$\text{Minimize } Z = x_1 - 2x_2 - 3x_3$$

$$\text{subject to constraints: } -2x_1 + x_2 + 3x_3 = 2; 2x_1 + 3x_2 + 4x_3 = 1; x_1, x_2, x_3 \geq 0$$
