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2603E1329

Candidate's Seat No :________

B.Sc. Semester-5 Examination
CC 303
Mathematics
March-2024

Time : 2-30 Hours] **[Max. Marks : 70**

Instructions: 1) All questions are compulsory.
2) Figures to the right indicate full marks of the question/sub-question.
3) Notations used in this question paper carry their usual meaning.

Q-1 *a)* State and prove triangular inequality. Hence prove that, if z_1 and z_2 are complex numbers then $|z_1 + z_2| \leq |z_1| + |z_2|$. **(07)**

b) Find all values of $(-8i)^{\frac{1}{3}}$. **(07)**

OR

Q-1 *a)* Suppose that $z_n = x_n + iy_n (n = 1, 2, 3, \ldots)$ and $z = x + iy$. Then prove that **(07)**

$$\lim_{n \to \infty} z_n = z \Leftrightarrow \lim_{n \to \infty} x_n = x \text{ and } \lim_{n \to \infty} y_n = y.$$

b) If $\sin(\alpha + i\beta) = x + iy$, then find x and y, and prove that **(07)**

(i) $\frac{x^2}{\cosh^2 \beta} + \frac{y^2}{\cosh^2 \beta} = 1$

(ii) $\frac{x^2}{\sin^2 \alpha} - \frac{y^2}{\cos^2 \alpha} = 1.$

Q-2 *a)* Prove that if a function $f(z) = u(x, y) + i\, v(x, y)$ is differentiable in D then u_x, u_y, v_x, v_y exist and $u_x = v_y$ and $u_y = -v_x$ in D. **(07)**

b) Define harmonic function. Find the Harmonic conjugate of $y^3 - 3x^2y$ and corresponding analytic function in terms of z. **(07)**

OR

Q-2 *a)* The function f is defined as $f(z) = \begin{cases} \frac{(\bar{z})^2}{z}; & z \neq 0 \\ 0; & z = 0 \end{cases}$, then show that $f(z)$ is not analytic at $z = 0$; even if it satisfies Cauchy-Riemann equations at the origin. **(07)**

b) If $f(z) = u(r, \theta) + i\, v(r, \theta)$ be an analytic function and $u = -r^3 \sin 3\theta$ then find a function $v(r, \theta)$ and also express the function $f(z)$ in terms of z. **(07)**

Q-3 *a)* Show that the mapping $w = \frac{1}{z}$ transforms the circles and lines into circles and lines. **(07)**

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b) Prove that the magnitude and the direction of angle between the lines $y = 2x$ and $y = x - 1$ remains same under the mapping $w = f(z) = z^2$. Sketch all curves and determine corresponding directions along them. **(07)**

OR

Q-3 a) Prove that if $f(z)$ is analytic at z_0 and $f'(z_0) \neq 0$, then $w = f(z)$ is conformal at z_0. **(07)**

b) Find the bilinear transformations which map **(07)**

(i) $-1,\ 0,\ 1$ of z −plane onto $-i,\ 1,\ i$ of w −plane respectively

(ii) $\infty,\ -i,\ 0$ of z −plane onto $0,\ 1,\ \infty$ of w −plane respectively.

Q-4 a) State and prove Bessel's inequality for the Fourier series. **(07)**

b) Obtain the Fourier series expansion of $f(x) = x \sin x$ in $[-\pi,\ \pi]$. Hence, deduce that $\frac{\pi}{4} = \frac{1}{2} + \frac{1}{1\cdot3} - \frac{1}{3\cdot5} + \frac{1}{5\cdot7} - \cdots$. **(07)**

OR

Q-4 a) State and prove Euler's formula for the Fourier coefficients. **(07)**

b) Find a sine series for the function $f(x) = x$ for $0 < x < \frac{\pi}{2}$ and $f(x) = 0$ for $\frac{\pi}{2} < x < \pi$. **(07)**

Q-5 **Answer the following questions in short (Any SEVEN)** **(14)**

1 Show that $|Im(1 - \bar{z} + z^2)| < 3$, when $|z| < 1$.

2 Prove that $|z| \leq |Re\ z| + |Im\ z|$.

3 Find the modulus and principal argument of the complex number $-\sqrt{3} - i$.

4 Is $u(x,\ y) = 2x(1 - y)$ harmonic?

5 Write the C-R equations and Laplace's equation in polar form.

6 Is the function $f(z) = |z|^2$ entire? Justify.

7 Find the critical points of the mapping $w = \frac{z^3}{3} - z$.

8 Find the non-conformal points of the mapping $f(z) = 2z^3 + 15z^2 - 6z + 9$.

9 Find the singular points of $|z|^2$ and $\frac{1}{z}$.

10 Obtain $\int_{-\pi}^{\pi} \cos^2 nx\ dx$, for all $n \in N$.

11 Obtain $\int_{-\pi}^{\pi} \cos nx\ dx$, for all $n \in N$.

12 True / False:

(i) The Fourier series of the even function contains only cosine terms.

(ii) The Fourier series of odd function contains only cosine terms.

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