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2509N509

Candidate's Seat No : ________

B.Sc Sem.-6 (Rep) Examination

CC 307

Mathematics

September-2024

Time : 2-30 Hours] **[Max. Marks : 70**

Q.1 (A) Define characteristic of a ring. Prove that the characteristic of an integral Domain is either prime number or zero. [7]

(B) In the set of all integers Z, the operations $\oplus$ and $\otimes$ are defined by $a \oplus b = a + b - 1$ and $a \otimes b = a + b - ab$ for all $a, b \in Z$ then show that $(Z, \oplus, \otimes)$ is a commutative ring with unity. Is an integral domain? Is it a field? [7]

OR

Q.1 (A) Prove that a field is an integral domain. Is converse true? Justify your answer. [7]

(B) Show that the set $F = \{a + b\sqrt{2} \,/\, a, b \in Q\}$ is a field under usual addition and multiplication defined as follows.
Addition : $(a + b\sqrt{2}) + (c + d\sqrt{2}) = (a + c) + (b + d)\sqrt{2}$ and
Multiplication: $(a + b\sqrt{2}) \cdot (c + d\sqrt{2}) = (ac + 2bd) + (ad + bc)\sqrt{2}$. [7]

Q.2 (A) State and prove the fundamental theorem on homomorphism ~~for ring.~~ ring [7]

(B) let R be the ring of all complex number and $R' = M_{2x2}(R) = \left\{ \begin{bmatrix} a & b \\ -b & a \end{bmatrix} / a, b \in R \right\}$ be a ring w.r.t usual addition and multiplication. Define $\phi : R \to R'$ by $\phi(a + ib) = \begin{bmatrix} a & b \\ -b & a \end{bmatrix}$; a+ib$\in$R then Verify whether ϕ is a homomorphism or not ? Is it an Isomorphism? More over find the Kernel of ϕ. [7]

OR

Q.2 (A) Define an ideal in a ring. State and Prove the necessary and sufficient condition for a non-empty subset of a ring to be an ideal of a ring. [7]

(B) Obtain all ideals of the ring $(Z_{18}, +_{18}, \bullet_{18})$ and prepare tables for co-responding quotient rings. [7]

Q.3 (A) State and prove the division algorithm for polynomials. [7]

(B) Define irreducible polynomial, Also find all rational roots of an equation $4x^5 + x^3 + x^2 - 3x + 1 = 0$. [7]

OR

Q.3 (A) For non-zero polynomial $f, g \in D[x]$ then in usual notation prove that [fg] = [f]+[g]. [7]

(B) Define g.c.d of two polynomials over a field F, Using Euclid's algorithm for the polynomials $f(x) = x^3 - 2x^2 + 3x - 7$ and $g(x) = x^2 + 2$ over the field R, find g.c.d of f(x) and g(x). Also express it into the form a(x)f(x)+b(x)g(x). [7]

P.T.O

Q.4 **(A)** Define maximal ideal. Prove that an ideal I in a commutative ring R with unity is a maximal ideal iff the quotient ring R/I is a field. [7]

(B) Prove that the ideal $I = <x^3 - x - 1>$ is a maximal ideal in $Z_3[x]$. [7]

OR

Q.4 **(A)** let R be a commutative ring with unity and I be an ideal of R then prove that R/I is an integral domain iff I is a prime ideal. [7]

(B) For an integral domain D and the field F, the mapping $\phi : D \to F$ defined by $\phi(a) = (a,1) : \forall a \in D$ where $F = \{[a,b]/(a,b) \in S, b \neq 0\}$ then show that $D \cong F$. [7]

Q.5 Answer the following in short (**ANY SEVEN**). [14]

(1) Give an example of a division ring.

(2) Is $(z_7, +_7, \cdot_7)$ integral domain ? Justify your answer.

(3) Give an example of ring without unity but it's subring with unity.

(4) Define Kernel of a homomorphism.

(5) If $I = 5Z$ is an ideal of the ring $R = (Z, +, \cdot)$ then write down all the elements in quotient ring R/I. Also, Solve equation $(I+2)\cdot X = I+3$ for $X \in R/I$

(6) Define principal ideal.

(7) Find f+g and fg for two polynomial f=(2,4,-2,4,1,0,3,0,0,0...)$\in Z[x]$ and g=(3,4,-1,2,1,5,0,0,0...)$\in Z[x]$.

(8) Obtain the quotient $q(x)$ and the remainder $r(x)$ on $f(x) = x^3 + 1$ dividing by $g(x) = x^2 + 3x - 5$ in $R[x]$.

(9) Define monic polynomial.

(10) Define an extension field and give an example of it.

(11) Find a polynomial with integer co efficient that has ½ ,2 and -1/3 as zeroes.

(12) Define prime ideal and Give an example of a prime ideal.

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