Seat No. : _______________

N13-109

November-2014

B.Sc., Sem.-V

STA-301: STATISTICS (Distribution Theory – I)

Time : 3 Hours] **[Max. Marks : 70**

Instructions : (1) **All** questions are compulsory and carry equal marks.
(2) Statistical tables and graph papers will be provided on request.
(3) Use of scientific calculator is allowed.

1. (a) For Geometric Distribution, state its probability mass function, mean and variance. A court is conducting a jury selection. Let X be the number of prospective jurors who will be examined until one is admitted as a juror for a trial. Suppose that X is a geometric random variable, and p, the probability of a juror being admitted, is 0.50. Find the mean and the standard deviation of X.

OR

For Negative binomial distribution, obtain the recurrent relation for the central moment.

(b) If a random variable X follows geometric distribution, then show that for any two positive integers m and n, $P [X > m + n \mid X>m] = P [X \geq n]$.

OR

An item is produced to large numbers. The machine is known to produce 5% defectives. A quality control inspector is examining the items by taking them at random. What is the probability that at least 4 items are to be examined in order to get 2 defectives ?

2. (a) With reference to the probability distribution theory, define the term : Truncation. Also, state it's different forms. Hence or otherwise, explain, in brief, truncation from left.

OR

Derive Truncated Binomial Distribution, truncated at X = 0. Hence or otherwise obtain the expression for its variance.

(b) Derive Truncated Poisson distribution, truncated at X = 0. Obtain its mean and variance.

OR

For a normal distribution with mean μ and standard deviation σ, derive the truncated normal distribution to the right of X = b.

3. (a) Define power series distribution. Derive the Binomial distribution and its m.g.f. as a special case of power series distribution.

OR

In usual notations, derive the recurrent relation for the central moments of power series distribution.

(b) For power series distribution, in usual notations, show that

$$\mu_1^{/} = \frac{\theta^2 f'(\theta)}{f(\theta)} \text{ and } \mu_2^{/} = \frac{\theta^2 f''(\theta)}{f(\theta)} + \frac{\theta f'(\theta)}{f(\theta)}$$

OR

For a Negative Binomial Distribution, using power series distribution, obtain first two cumulants.

4. (a) Define order statistics. State use of ordered statistics.

OR

If probability density function a random variable X is $f(x) = \begin{cases} 1, & 0 \le x \le 1 \\ 0, & \text{otherwise} \end{cases}$, then obtain the distribution of the smallest order statistics and a sample range.

(b) Obtain the distribution of the smallest and the largest order statistics.

OR

If probability distribution function of a random variable X is

$$F(x) = \begin{cases} 1 - e^{-x}, & 0 \le x \le 1 \\ 0, & \text{otherwise} \end{cases}$$

obtain the distribution of the largest order statistics and a sample range.

5. Answer the following questions, in brief :

(a) State the assumptions while deriving geometric distribution.

(b) What is the alternative representation of negative binomial distribution and to what experimental situation does it correspond ?

(c) State the moment generating function of negative binomial distribution and write the first two raw moments of it.

(d) State joint probability density function of order statistics. State the probability density function of r^{th} order statistics.

(e) Let $Y_1 < Y_2 < Y_3 < Y_4 < Y_5$ be the order statistics of 5 independent observations of an exponential distribution with mean 3. Compute the probability that Y_4 is less than 5.

(f) State an appropriate probability distribution for an attempt a three-point shot in basketball until you make a basket. State mean and variance of this probability distribution.

(g) Bob is a high-school basketball player. He is a 70% free throw shooter. During the season, what is the probability that Bob makes his third free throw on his fifth shot ?