2204M197

Candidate's Seat No :________

Integ. FY M.Sc (CA&IT) (O) Examination
Matrix Algebra & Graph Theory
April-2024

Time : 2-00 Hours] **[Max. Marks : 70**

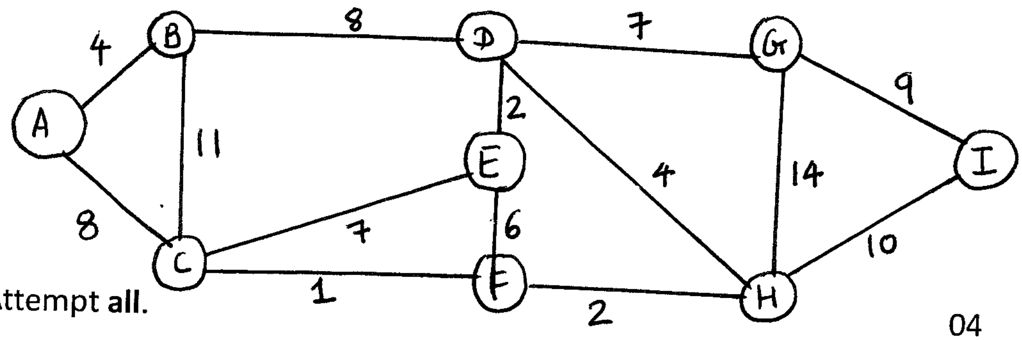
Instruction: Use of simple calculator is allowed.

Q-1 (a) Attempt any **two.** 10

(1) State and prove hand-shaking lemma and further prove that in any Graph G, there is an even number of odd vertices.

(2) Prove that there does not exist a party of seven people where each person know exactly seven of others.

(3) Use Kruskal's algorithm to calculate the minimal spanning tree from the following graph.



(b) Attempt **all.** 04

(1) Define edge deleted subgraph.

(2) Define radius of a graph.

(3) Define connected graph graph.

(4) Give an example of 2- regular graph on three vertices.

Q-2 (a) Attempt any **two.** 10

(1) If $A = \begin{pmatrix} -1 & -2 & 1 \\ 2 & 3 & 4 \\ 1 & -3 & 4 \end{pmatrix}$, and $B = \begin{pmatrix} -3 & 4 & -1 \\ 3 & -5 & 6 \\ \frac{2}{5} & 1 & \frac{-1}{7} \end{pmatrix}$ then check whether $A^2 - 2AB + B^2 = (A-B)^2$ or not?

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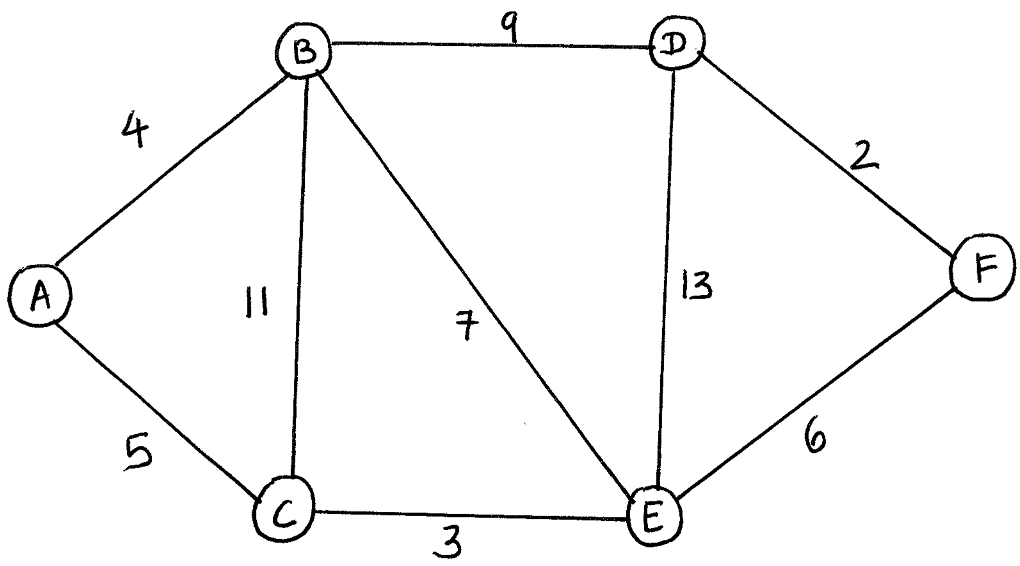
(2) Define Planar graph.
Prove that $K_{3,3}$ is not a planar graph.

(3) Apply Dijsktra algorithm to find the shortest path between A to F.



(b) Attempt **any four.** 04

(1) Define bipartite graph.

(2) Define Tree.

(3) Define isomorphism of graphs.

(4) Draw complete graph on four vertices.

(5) Give an example of a bipartite graph which is not a complete Bipartite graph.

Q-3 (a) Attempt any **two.** 10

(1) Find rank form of a matrix $A = \begin{pmatrix} 3/7 & -2 & 3 & 1 & 4 \\ 2 & 3 & -1 & 3 & -1 \\ 1 & -1 & -1/5 & 2 & -1 \end{pmatrix}$.

(2) Find the solution (if possible) of the following system of linear equations

$$x + y - z = 1,$$
$$x - y + z = 2,$$
$$-x + y + z = 3.$$

(3) Find inverse of a matrix $A = \begin{pmatrix} 1 & 0 & 3 \\ 3 & 4 & 1 \\ 4 & 0 & -1 \end{pmatrix}$ (if possible).

(b) Attempt **all**. 04

(1) If $A = \begin{pmatrix} 3 & -4 & 1 \\ 7 & -2 & 10 \\ 3 & -4 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} -2 & 3 & 1 \\ 5 & -7 & 6 \\ 3 & 15 & -12 \end{pmatrix}$ then calculate 4A-5B.

(2) Find characteristic polynomial of a matrix $A = \begin{pmatrix} 3 & 4 \\ 1 & -2 \end{pmatrix}$.

Q-4 (a) Attempt any **one**. 10

(1) Prove that a mapping $T: R^3 \to R^4$ defined by $T(x, y, z) = \left(\frac{x-y+z}{3}, \frac{x+y-z}{3}, \frac{z}{3}, \frac{-x+y+z}{3}\right)$ is a linear map.

(2) Prove that a set $U = \{(1,0,1),(0,1,1),(1,1,0)\}$ forms a Basis of R^3.

(b) Attempt **any two.** 04

(1) Define vector space.

(2) Write a vector (3, -1/2, 7/2) as a linear combination of vectors {(2, -1, 3), (3, 2, 1), (1, 2, 1)}.

(3) Define (1) Linear independence

(2) Linear dependence

Q-5 (a) Attempt any **two.** 10

(1) How many permutations are possible using all letters of the word **"HEXAGON"**? In dictionary order of these words, which place Will this word occupy?

(2) Find a linear transformation such that $T: R^4 \to R^3$ such that
T (1, 2, 3) = (1, 2, 3, 0),
T (2, 3, 4) = (2, 3, 4, 0) and
T (-1, 7, 5) = (2, 1, 3, 5). Also, find T (3/2, 1/5, -1/2).

(3) Let $T: R^3 \to R^3$ be a linear transformation defined by $T(x, y, z) = (-x + y - z, x + 2y - 3z, x - 3y + 4z)$. Find Ker(T).

(b) Attempt **all.** 04

(1) Define linear transformation.

(2) Define kernel of a linear transformation.

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