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2203N1708

Candidate's Seat No :________

M.Sc. Semester-3 Examination

503-EA

Mathematics

March-2024

Time : 2-30 Hours] **[Max. Marks : 70**

1. (A) Let $f(x,y,z) = x^2e^{x-y+3z}$. Compute the differential df.
   Use the differential to estimate the difference $f(1.1, 1.2, -0.1) - f(1,1,0)$. **7**

   (B) Find the extreme values of the function $f(x,y) = 2x^2 - 3y^2 + 2y$ on the set $S = \{(x,y) : x^2 + y^2 \leq 3\}$. **7**

OR

1. (A) Find and classify the critical points of the function $f(x,y) = xy(8 - 2x - 4y)$. **7**

   (B) Let $(u,v) = \mathbf{f}(x,y,z) = (xyz^2 - 4y^2, 3xy^2 - yz)$.
   Compute $D\mathbf{f}(x,y,z)$, $\partial(u,v)/\partial(x,y)$, $\partial(u,v)/\partial(y,z)$, and $\partial(u,v)/\partial(x,z)$. **7**

2. (A) Investigate the possibility of solving the equations **7**

$$\begin{cases} xz + 2xy - 3yz = 0 \\ xyz - y + z = 1 \end{cases}$$

   for two of the variables as functions of the third near the point $(x,y,z) = (1,1,1)$.

   (B) Find an equation for the tangent plane to the following parametrized surface at the point $(1, -2, 1)$:
   $x = e^{u-v}$, $y = u - 3v$, $z = \frac{1}{2}(u^2 + v^2)$ **7**

OR

2. (A) Let $\mathbf{F}(x,y) = x^2 + 7y^2 - 7$.
   Determine whether the set $S = \{(x,y) : \mathbf{F}(x,y) = 0\}$ is a smooth curve. Draw a sketch of S. Examine the nature of S near any point where $\Delta\mathbf{F} = \mathbf{0}$. Near which points of S is S the graph of a function $y = f(x)$? $x = f(y)$? **7**

   (B) Let $(u,v) = \mathbf{f}(x,y) = (x^2 + 2xy + y^2, 2x + 2y)$.
   Compute the Jacobian $\det D\mathbf{f}$. Draw a sketch of the images of some of the lines $x =$ constant and $y =$ constant in the $uv-$plane. Find a formula for a local inverse of $\mathbf{f}$. **7**

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3. (A) Evaluate $\iint_S (x + 3y^3)dA$, S = the upper half ($y \geq 0$) of the unit disc $x^2 + y^2 \leq 1$. **7**

   (B) Find the centroid of the tetrahedron bounded by the coordinate planes and the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$. **7**

OR

3. (A) Find the area of the region inside the cardioid $r = 1 + \cos\theta$ (polar coordinates). **7**

   (B) Find the mass of a ball of radius R if the mass density is c times the distance from the boundary of the ball. **7**

4. (A) Compute $\int_C \mathbf{F} \cdot d\mathbf{x}$,
   where $\mathbf{F}(x, y) = (x^2y, x^3y^2)$ and C is the closed curve formed by portions of the line $y = 4$ and the parabola $y = x^2$, oriented counterclockwise. **7**

   (B) Let S be the annulus $1 \leq x^2 + y^2 \leq 4$.
   Compute $\int_{\partial S}(xy^2dy - x^2ydx)$ by using the Green's theorem. **7**

OR

4. (A) Find the centroid of the upper hemisphere of the unit sphere $x^2 + y^2 + z^2 = 1$. **7**

   (B) Evaluate $\iint_S \mathbf{F} \cdot \mathbf{n}\, dA$, where $\mathbf{F}(x, y, z) = xy\mathbf{i} + z\mathbf{j}$ and S is the the triangle with vertices $(2, 0, 0), (0, 2, 0), (0, 0, 2)$, oriented so that the normal points upward. **7**

5. **Attempt any seven of the following.** 14

   (1) The directional derivative of the function $f(x, y) = x^2 - xy + 3y^2$ at the point $(-1, 2)$ in the direction $(\frac{3}{5}, \frac{4}{5})$ is

   (A) -8 (C) $64/5$

   (B) 8 (D) $52/5$

   (2) z is determined as a function of y and x by the equation $x + y^2 + z^3 = 3xyz$. Which of the following are true?

   (A) $\frac{\partial z}{\partial x} = \frac{3z^2 - 3xy}{3yz - 1}$ (C) $\frac{\partial z}{\partial x} = \frac{3yz - 1}{3z^2 - 3xy}$

   (B) $\frac{\partial z}{\partial y} = \frac{3xz - 2y}{3z^2 - 3xy}$ (D) $\frac{\partial z}{\partial y} = \frac{3z^2 - 3xy}{3z - 2y}$

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(3) The tangent plane to the surface $z = x^2 - y^3$ at the point $(2, 1, 3)$ is

(A) $4x - 3y - z = -1$ (C) $4x - 3y - z = 2$

(B) $4x - 3y - z = -2$ (D) $4x + 3y - z = -14$

(4) The absolute maximum value of $f(x, y) = x^2 + y^2 + y$ on the disc $x^2 + y^2 \leq 1$ is

(A) 2 (C) 1

(B) 3 (D) -2

(5) The curve $\mathbf{f}(t) = (\cos t, \sin t)$ is a

(A) circle (C) ellipse

(B) parabola (D) straight line

(6) Let $\mathbf{f}(t) = ((t^2 - 1)/(t^2 + 1), t(t^2 - 1)/(t^2 + 1))$. Then $S = \{\mathbf{f}(t) : t \in \mathbb{R}\}$ is the locus of the equation

(A) $y(1 - x) = x(1 + x)$

(B) $y^2(1 - x) = x^2(1 + x)$

(C) $y^2(1 + x) = x^2(1 - x)$

(D) $y^2(1 - x) = x(1 + x)$

(7) Let a transformation be given by $u = e^x \cos y, v = e^x \sin y$. Which of the following describe the image of line $x = 5$?

(A) The image is an ellipse

(B) The image is a straight line

(C) The image is a hyperbola

(D) The image is a circle

(8) Let S be the region in the left half plane between the curve $y = x^3$ and the line $y = 4x$. Which of the following iterated integrals express the double integral $\iint_S f dA$?

(A) $\int_{-2}^{0} \int_{4x}^{x^3} f(x, y)\, dy\, dx$ (C) $\int_{-2}^{0} \int_{4x}^{x^2} f(x, y)\, dy\, dx$

(B) $\int_{-8}^{0} \int_{y^{1/3}}^{y/4} f(x, y)\, dx\, dy$ (D) $\int_{-8}^{0} \int_{y^{1/3}}^{y/3} f(x, y)\, dx\, dy$

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(9) The value of the double integral $\iint_R y dA$, where $R = [0, 2] \times [0, 1]$ is

(A) 5

(B) 3

(C) 2

(D) 1

(10) The arc length of the parametrized curve $\mathbf{g}(t) = (\frac{1}{3}t^3 - t, t^2), 0 \leq t \leq 2$, is

(A) 8/3

(B) 9

(C) 14/3

(D) 10/3

(11) Let $\mathbf{F}(x, y, z) = x^2 z\mathbf{i} + 4xyz\mathbf{j} + (y - 3xz^2)\mathbf{k}$. Then div $\mathbf{F}$ equals

(A) $12xz$

(B) $6xz$

(C) $4xz$

(D) 0

(12) Let $\mathbf{F}$ and $\mathbf{G}$ be vector fields, then $\text{div}(\mathbf{F} \times \mathbf{G}) =$ __________.

(A) $\mathbf{G} \cdot (\text{curl } \mathbf{F}) - \mathbf{F} \cdot (\text{curl } G)$

(B) $\mathbf{F} \cdot (\text{curl } \mathbf{G}) - \mathbf{G} \cdot (\text{curl } F)$

(C) $\mathbf{G} \cdot (\text{curl } \mathbf{F}) + \mathbf{F} \cdot (\text{curl } G)$

(D) $\mathbf{F} \cdot (\text{curl } \mathbf{G}) + \mathbf{G} \cdot (\text{curl } F)$

2203N1708-5 Candidate's Seat No :________

M.Sc. Semester-3 Examination

503-EB

Mathematics

Time : 2-30 Hours] **March-2024** **[Max. Marks : 70**

Instructions: All questions are compulsory. Use of non-programmable scientific calculator is allowed.

Q.1 **(a)** A 7-year bond with Rs.1000 face value has a annual coupon rate of 5%. The current market rate of interest is 5.5%. Calculate the Macaulay duration. **(07)**

OR

(a) Consider an investment that has the following expected cash flows: **(07)**

Year	Cash Flows (Rs.)
Today	-10,000
1	1,000
2	1,000
3	9,000

What is the net present value and profitability index on this investment? The discount rate is 5%.

(b) Explain in detail the comparison of NPV and IRR with suitable example. **(07)**

OR

(b) Explain in brief the Macaulay duration and write an expression which establishes the relation between Macaulay and Modified duration. **(07)**

Q.2 **(a)** Find Treynor Ratio and explain which manager is preferable and why? **(07)**

Managers	**Average Annual Return**	**Beta**
Manager A	10%	0.90
Manager B	14%	1.03
Manager C	15%	1.20

OR

(a) Explain in detaii: The Theory of Capital Asset Pricing Model (CAPM) **(07)**

(b) Explain Security Market Line (SML) and Capital Market Line (CML) with graphical representation. **(07)**

OR

(b) Explain the Markowitz theory indicating its assumptions, efficient portfolio, efficient frontier, and the limitations of the theory. **(07)**

Q.3 **(a)** In usual notations explain and derive the derivative price formula by the method of Replicating portfolios. Also, explain delta hedging. **(07)**

OR

(a) Write a note on short hedge and explain how one can reduce risk of loss by using short hedge? **(07)**

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(b) Find the value of American put option for the following data by using two-step binomial model. $S_0 = 20, X = 21, u = 1.1, d = 0.9, r = 0.12, T = 0.25$ **(07)**

OR

(b) Define options. Explain its types with suitable example. **(07)**

Q.4 **(a)** Derive the Black-Scholes-Merton formula for a European put option by using the put-call parity formula. **(07)**

OR

(a) Consider a call/put option on a non-dividend paying stock where the current stock price is \$50, the strike price \$55, the risk-free interest rate is 5% per annum with continuously compounding, the time to maturity is 20 weeks and volatility is 35%. Find the Gamma of call/put option. **(07)**

(b) Using the following data, compute the price of the associated European call option by Black-Scholes formula. $S_0 = 1500, X = 1650, r = 0.065, T = 6$ month, $\sigma = 0.30$ (Use the tabulated value: $N(d_1) = 0.4246$, $N(d_2) = 0.3438$) **(07)**

OR

(b) Explain in detail the two equations given below and all the terms within them. **(07)**

$$dS = \mu S dt + \sigma S dB$$

$$S_t = S_0 \exp\left[\sigma B_t + \left(\mu - \frac{\sigma^2}{2}\right)t\right]$$

Q.5 Attempt any **SEVEN** out of **TWELVE:** **(14)**

(1) What is Present Value and Future Value for an Annuity?
(2) State interpretation of Profitability Index.
(3) What is accrued interest?
(4) Explain: Diversification
(5) Define: Efficient Frontier
(6) According to Markowitz, Investor attitudes toward portfolio depends exclusively on (i) .... (ii) ....
(7) Write down the formula for Greek: Vega
(8) Define: Implied Volatility
(9) What is the basic difference between Forward and Futures contracts?
(10) Write the expression representing expected value and Variance of Log-normal distribution.
(11) Define: Efficient Frontier
(12) Define: Yield Curve

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