**2203E1295** Candidate's Seat No :_____

### B.Sc. Semester-5 Examination
### CC 302
### Mathematics

Time : 2-30 Hours]　　　　　　　　March-2024　　　　　　　[Max. Marks : 70

**Instructions:** (1) All questions are compulsory.
　　　　　　　(2) Figure to the right indicates full marks of the question.

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|---|---|---|---|
| **Q-1** | **(a)** | Prove that there exists a positive real number $x$ such that $x^2=2$. | **(7)** |
| | **(b)** | Find all $x \in \mathbb{R}$ that satisfy both $|2x-3| <5$ and $|x+1|>2$ simultaneously. | **(7)** |

**OR**

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|---|---|---|---|
| **Q-1** | **(a)** | If A is any set, then prove that there is no surjection of A onto the set $P(A)$ of all subsets of A. | **(7)** |
| | **(b)** | Prove that $\sqrt{2}$ is not a rational number. | **(7)** |
| **Q-2** | **(a)** | If $(x_n)$ and $(y_n)$ are convergent sequences and $\lim y_n \neq 0$ then prove that | **(7)** |

$$\lim\left(\frac{x_n}{y_n}\right) = \frac{\lim x_n}{\lim y_n}$$

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| | **(b)** | Define sequence$\{x_n\}$ inductively by $x_1=5$ and $x_{n+1}=\sqrt{4+x_n}, n \geq 1$. Prove that$\{x_n\}$ converges and determine the limit. | **(7)** |

**OR**

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|---|---|---|---|
| **Q-2** | **(a)** | State and prove Bolzano-Weierstrass theorem. | **(7)** |
| | **(b)** | Let $x_n = \sum_{k=1}^{n}\frac{1}{k} = 1+\frac{1}{2}+\frac{1}{3}+... +\frac{1}{n}$. Show that sequence $\{x_n\}$ is not Cauchy, not convergent and not bounded sequence. | **(7)** |

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| **Q-3** | **(a)** | Suppose that the function $f$ is continuous on the interval [a,b]. Then prove that $f$ attains its maximum and minimum values on [a,b] . | **(7)** |
| | **(b)** | Find limit using the definition: $\lim_{x \to 1}\frac{1}{5x-3}$ | **(7)** |

**OR**

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|---|---|---|---|
| **Q-3** | **(a)** | State and prove Intermediate Value Theorem(IVT). | **(7)** |
| | **(b)** | Discuss the uniform continuity of the following functions: <br> (i) *f(x)= 1/x* on [c,∞) , c > 0 <br> (ii) *g(x)= sin(2π/x)* on (0,1) | **(7)** |

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| **Q-4** | **(a)** | State and prove Mean Value Theorem. | **(7)** |
| | **(b)** | Evaluate: (1) $\lim_{x \to \infty}\frac{x^3}{e^x}$ (2) $\lim_{x \to 0^+}\frac{\tan x - x}{x^3}$ $x \in (0, \pi/2)$ | **(7)** |

**OR**

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|---|---|---|---|
| **Q-4** | **(a)** | State and prove L'Hospital's First Rule. | **(7)** |
| | **(b)** | If $3a-4b+6c-12d=0$, then prove that one root of cubic equation $ax^3+bx^2+cx+d=0$ lies between -1 and 0. | **(7)** |

[ P . T . O ]

**Q-5**      Attempt any Seven short questions:      **(14)**

**(i)**      Find the lubA and glbA of the set $A = \left\{ \dfrac{5n+2}{n} \; / \; n \in N \right\}$.

**(ii)**      If $B = \{ x \in Q : \; x^2 < 2 \}$ then $inf\, B =$ _____

**(iii)**      Give example of a set which contains its *lub* but does not contains its *glb*.

**(iv)**      Find the cluster points of the sequence $\{x_n\} = \{ cos(n\pi/3) \}$

**(v)**      Give an example of a sequence which is bounded and has finite range.

**(vi)**      Give an example of a Cauchy sequence.

**(vii)**      Evaluate: $\lim\limits_{x \to 1^+} [x-2] + [x+4]$ if exist.

**(viii)**      Define Uniform continuity.

**(ix)**      Give example of a function with removable discontinuity.

**(x)**      State Darboux's theorem.

**(xi)**      Determine where the function $f(x) = |x^2 - 8|$ is not differentiable.

**(xii)**      Give an example of function which is continuous and not differentiable.

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