2103E1261 Candidate's Seat No :________

B.Sc. Semester-5 Examination

CC 301

Mathematics

March-2024

2-30 Hours] **[Max. Marks : 70**

n : (i) All the questions are compulsory and carry equal marks.

(ii) Notations are usual everywhere.

(iii) The right hand side figures indicate marks of the question/sub-question.

T : U→V is linear map , $v_0 \in R(T)$ and if $T(u) = \bar{0}_V$ has only a trivial solution $= \bar{0}_U$ then prove that the operator equation $T(u) = v_0$ has a unique solution. [7]

hat the 2 × 2 determinant function $f: R^2 \to R$ defined by
$) = x_1y_2 - x_2y_1$ for $x = (x_1, x_2)$, $y = (y_1, y_2) \in R^2$ is a bilinear form. [7]

OR

nd Prove the Dual Basis Existence Theorem. [7]

e dual basis of the basis B = {(1,0,0), (1,1,0), (1,1,1)} for the vector space V_3 . [7]

ove that an orthogonal set of nonzero vectors is linearly independent. [7]

or $x = (x_1, x_2), y = (y_1, y_2) \in R^2$ the map $< , >$ is defined as
$> = x_1y_1 - x_1y_2 - x_2y_1 + 2x_2y_2$ then show that $< , >$ is an inner product on R^2 . [7]

OR

and Prove the Cauchy-Schwarz inequality. [7]

the Gram-Schmidt orthogonalization process to the basis
{(1, 1) (1, 2) } in order to get the orthonormal basis for V_2 . [7]

$\neq j$, $\alpha \in R$ and if *det* : $V^n \to R$ is a function satisfying the expected properties
the determinant then prove the followings :

i) $det(v_1, v_2, \dots, v_i, \dots, v_j, \dots, v_n) = det(v_1, v_2, \dots, v_i + \alpha v_j, \dots, v_j, \dots, v_n)$

ii) $det(v_1, v_2, \dots, v_i, \dots, v_j, \dots, v_n) = -det(v_1, v_2, \dots, v_j, \dots, v_i, \dots, v_n)$. [7]

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If A= $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 0 & 6 & 7 \\ 0 & 8 & 9 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ then compute $det A$ without expansion. [7]

OR

ͤ the formula of finding area of a parallelogram in terms of a 2 × 2 determinant. [7]

he Cramer's rule to solve : $2x - 3y + z = 1$

$x + y - z = 0$

$x - 2y + z = -1.$ [7]

ːxpress the characteristic equation of 2×2 matrix in terms of its trace and determinant.

.lso prove that a 2×2 real and symmetric matrix has only real eigen values. [7]

ınalize the matrix $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \\ 1 & 0 & 2 \end{bmatrix}$. [7]

OR

and Prove the Caley-Hamilton's Theorem. [7]

fy the quadric in R^3 given by $f(x, y, z) \equiv 4xz + 4y^2 + 8y + 8 = 0.$ [7]

npt any **SEVEN** of the followings in **Short** : [14]

efine a linear functional and the Dual Space of a vector space.

efine homogeneous and nonhomogeneous operator equations.

efine a Bilinear form and an Annihilator.

efine a Euclidian Space and a Unitary space.

efine orthogonal projection of a vector along a nonzero vector.

ͤfine an orthogonal linear map and orthogonal complement of a subspace of ips V.

ate any two expected properties of the determinant function.

ːate the Laplace expansion for finding the value of a determinant.

ıd detA without expansion if $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 4 & 5 & 6 \end{bmatrix}$

efine an eigen value and eigen vector of an endomorphism.

ͤfine a symmetric linear map and a quadric.

ıte the spectral theorem.

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