2011N767

Candidate's Seat No : ________

IM.Sc AIML & DS Sem.-5 Examination

CC 301

Differential Equations

Time : 2-30 Hours] **November-2024** **[Max. Marks : 70**

Instructions: All questions are compulsory. Use of non-programmable scientific calculator is allowed.

Q.1 **(a)** Find the differential equation of the family of circles of radius r whose center lies on the x-axis. (07)

(b) Solve differential equation $\frac{dy}{dx} = e^{x-y} + x^3 e^{-y}$, using variable separable method. (07)

OR

(a) Find the orthogonal trajectories of the family of curve $\frac{x^2}{a^2} + \frac{y^2}{b^2+\lambda} = 1$, where λ is a parameter. (07)

(b) Solve the homogeneous linear differential equation $y'' - 4y' + 4y = 0$, where $y(0) = 3,\ y'(0) = 1$. (07)

Q.2 **(a)** Using Lagrange's method, solve equation $pz - qz = z^2 + (x+y)^2$. (07)

(b) Find the complete solution of equation $p + pq = qz$. (07)

OR

(a) Solve the non-linear PDE $p + q^2 = 1$. (07)

(b) Form the partial differential equation by eliminating the arbitrary function from the equation $xyz = \phi(x+y+z)$. (07)

Q.3 **(a)** Using Charpit's method, solve $px + qy = pq$. (07)

(b) Find the complementary function (C.F.) of $\frac{\partial^3 z}{\partial x^3} - 3\frac{\partial^3 z}{\partial^2 x \partial y} + 2\frac{\partial^3 z}{\partial y^3} = 0$ (07)

OR

(a) Solve non-homogeneous linear PDE $(D^2 - DD' + D' - 1)z = \cos(x+2y) + e^y$. (07)

(b) Discuss the classification of second order linear partial differential equations. (07)

Q.4 **(a)** Using Euler's method find the value of y for $\frac{dy}{dx} = x + y, y(0) = 1$ when $x = 0.1, 0.2$ with step size $h = 0.05$. (07)

(b) Obtain the trapezoidal formula. (07)

OR

(a) Derive the Simpson's 1/3 formula. (07)

(b) Using modified Euler's method find the value of y for $\frac{dy}{dx} = 1 - y, y(0) = 0$ when $x = 0.1, 0.2$. (07)

(P.T.O)

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Q.5 Attempt any **SEVEN** out of **TWELVE:** (14)

(1) Define and give an example of order and degree of a differential equation.

(2) Solve the non-homogeneous linear differential equation $(4D^2 - 4D + 1)y = e^{\frac{x}{2}}$.

(3) Find complementary function and particular integral of equation $(D^2 + 9)y = \sin 4x$.

(4) Define partial differential equation and solve the PDE $\frac{\partial^2 u}{\partial x \partial t} = e^{-t} \cos x$.

(5) Form the PDE $z = (x - 2)^2 + (y - 3)^2$.

(6) Solve the Clairaut equation $z = px + qy + p^2 q^2$.

(7) Solve homogeneous linear partial differential equation $r - s - 6t = 0$.

(8) Is the PDE $\frac{\partial^2 u}{\partial t^2} + 4\frac{\partial^2 u}{\partial x \partial t} + 4\frac{\partial^2 u}{\partial x^2} = 0$ elliptic?

(9) Write-down Heat equation, Wave equation and Laplace equation.

(10) Derive the Newton–Cotes formula.

(11) Discuss the fourth order Runge-Kutta method.

(12) Using the second order Runge-Kutta method find an approximate value of y given that $\frac{dy}{dx} = x - y^2$ and y (0) =1 at $x = 0.2$ taking $h = 0.1$.

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