

M.Sc Sem-3 Examination

501

Statistics

Time : 2-30 Hours]

November-2024

[Max. Marks : 70

Note: Attempt all questions.

Q.1

- (i) Define Test Function, Randomised Test Function and Non-Randomised Test Function. [7]
 (ii) State and prove N-P fundamental lemma for randomized test. [7]

OR

- (i) Let $X \sim$ Binomial distribution. Check whether distribution possess MLR property in $T(X)$ or not. [7]
 (ii) Define unbiased test. Show that test obtained by N-P lemma is always unbiased. [7]

Q.2

- (i) Define Likelihood Ratio Test . Under what circumstances would you recommend this test? State its properties. [7]
 (ii) Let X_1, X_2, \dots, X_n be a random sample from $U(0, \theta)$, $\theta > 0$. Obtain $(1 - \alpha) 100\%$ UMA CI for θ . [7]

OR

- (i) Let X_1, X_2, \dots, X_n be a random sample from $N(\mu, \sigma^2)$, where σ^2 is known. Develop the likelihood ratio test for testing $H_0 : \mu = \mu_0$ (specified) against $H_1 : \mu < \mu_0$. [7]
 (ii) Let X_1, X_2, \dots, X_n be a random sample from $f(x, \theta) = e^{-(x-\theta)}$, $x > \theta$. Obtain $(1 - \alpha) 100\%$ UMA CI for θ . [7]

Q.3

- (i) Develop the S.P.R.T. for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1 (> \theta_0)$, where θ is the parameter of a Poisson distribution. Find approximate expressions for OC function and ASN function of the test. [7]
 (ii) Develop the S.P.R.T. for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1 (> \theta_0)$, based on a random sample of size n from a population with p.d.f.

$$f(x, \theta) = \frac{1}{\theta} e^{-x/\theta}, x > 0, \theta > 0$$

Also obtain ASN and OC functions.

[7]

OR

- (i) Develop the S.P.R.T. for testing $H_0 : \pi = \pi_0$ against $H_1 : \pi = \pi_1$, based on a random sample from a binomial population with parameters (n, π) , n being known. Obtain its OC and ASN functions. [7]
- (ii) Give the S.P.R.T. for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1 (> \theta_0)$, in sampling from a normal density

$$f(x, \theta) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x-\theta}{\sigma}\right)^2\right], \quad -\infty < x < \infty$$

where σ is known. Also obtain its OC function. [7]

Q.4

- (i) Explain Kolmogorov-Smirnov one-sample test. [7]
- (ii) Explain Kruskal-Wallis one-way ANOVA test. [7]

OR

- (i) Explain Kolmogorov-Smirnov two-sample test. [7]
- (ii) Explain Friedman's two-way analysis of variance test. [7]

Q.5 Answer any seven: [14]

- (i) Neyman-Pearson lemma is used for finding most powerful test for
- | | |
|------------------------------------|---------------------------------------|
| (A) simple vs simple hypothesis | (B) simple vs composite hypothesis |
| (C) composite vs simple hypothesis | (D) composite vs composite hypothesis |
- (ii) Define UMPU test.
- (iii) Give an example of simple hypothesis.
- (iv) Define UMA confidence set.
- (v) A structure determined by a statistic that is independent of a sufficient statistic is known as _____.
- (vi) Test with Neyman structure is a _____
- | | |
|----------------------------------|-----------------------------|
| (A) Similar test | (B) subset of similar tests |
| (C) not a subset of similar test | (D) none of these |
- (vii) In SPRT α and β are _____.
- (viii) In Wald's SPRT sample size is _____.
- (ix) Sequential Probability Ratio Test of the general linear hypothesis terminates with probability
- | | | | |
|-------|---------|-------|-----------------------|
| (A) 0 | (B) 0.5 | (C) 1 | (D) none of the above |
|-------|---------|-------|-----------------------|
- (x) The Kruskal-Wallis test is the non parametric alternative to the _____.
 (A) Factorial design (B) One-way ANOVA (C) Two-way ANOVA (D) none of the above
- (xi) Give one advantage of non parametric test.
- (xii) Compare to parametric tests, the non parametric tests
- | | |
|----------------------------|-----------------------------|
| (A) less accurate | (B) less efficient |
| (C) computationally easier | (D) (B) and (C) but not (A) |
