2004N353

Candidate's Seat No :_____

M.Sc Semester-2 Examination

AMS 410

Mathematical Methods

April 2024

Time : 2-30 Hours] **[Max. Marks : 70**

Instructions: All questions are compulsory. Use of non-programmable scientific calculator is allowed.

Q.1 **(a)** Using modified Euler's method, determine the value of y when $x = 1.1$, correct up to four decimal places by taking $h = 0.05$. Given that $y(1) = 1$ and $\frac{dy}{dx} = y^2 - \frac{y}{x}$. (07)

(b) Determine the value of $y(2)$ using Milne's predictor – corrector method for $\frac{dy}{dx} = \frac{x+y}{2}$ and given that $y(0) = 2, y(0.5) = 2.636, y(1) = 3.595, y(1.5) = 4.968$. (07)

OR

(a) Solve the following differential equation using Heun's method. (07)
$\frac{dy}{dx} = xy$ with $y(1) = 2$ for $x = 1.4$ by taking $h = 0.2$, correct up to four decimal places.

(b) The velocity v of a particle at distance s from a point on its linear path is given by the following table: (07)

$s\ (m)$	0	2.5	5.0	7.5	10.0	12.5	15.0	17.5	20.0
$v\ (m/sec)$	16	19	21	22	20	17	13	11	9

Estimate the time taken by the particle to traverse the distance of 20 meter, using Simpson's $\frac{3}{8}$ rule.

Q.2 **(a)** Find the inverse Laplace transform of $\frac{3s-8}{4s^2+25}$. (07)

(b) Using the Laplace transform, find the solution of the initial value problem (07)
$$y'' + y = 0, y(0) = 1, y'(0) = -1$$

OR

(a) Obtain the Laplace transform of $t^2\, e^{-2t} \cos t$. (07)

(b) Find the $h(t)$ by the convolution theorem for $H(s) = \frac{1}{s\,(s^2+4)}$. (07)

Q.3 **(a)** Find the Fourier series of $f(x) = x^2, -2 < x < 2$. Also, Deduce that (07)
$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots = \frac{\pi^2}{12}$

(b) Solve using Fourier sin transform $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ subject to conditions (i) $u(x,t)$ is bounded, (07)
(ii) $u(0,t) = 0, t > 0$ (ii) $u(x,0) = \begin{cases} 1, & 0 < x < 1 \\ 0, & x \geq 1 \end{cases}$.

OR

(a) Find the Fourier cosine integral of the function $f(x) = \begin{cases} x, & 0 < x < 1 \\ 2 - x, & 1 < x < 2 \\ 0, & x > 2 \end{cases}$ (07)

(b) Find the half range Fourier cosine series for $f(x) = \sin x, x \in (0, \pi)$. Also derive that (07)
$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$$

P.T.O

Q.4 (a) Find and graph all sixth roots of unity in the complex plane. **(07)**

(b) Find the analytic function $w = u + iv$ whose imaginary part is given by $v = e^x(x \sin y + y \cos y)$. **(07)**

OR

(a) Find the image of $2x + y - 3 = 0$ under the transformation $w = z + 2i$. **(07)**

(b) Show that the transformation $w = \frac{1}{z}$ maps a circle in the z-plane into a circle or into a straight line in the w-plane. **(07)**

Q.5 Attempt any **SEVEN** out of **TWELVE:** **(14)**

(1) Define: Trapezoidal rule

(2) Using Euler's method.
find $y(0.2)$ given $\frac{dy}{dx} = 1 - 2xy, y(0) = 0$ with $h = 0.2$.

(3) Define: An Initial Value Problem

(4) Define: Unit Step Function

(5) Find the Laplace transform of $\sinh at$.

(6) Define: Second Shifting Theorem

(7) Find the Fourier transform of e^{3x}.

(8) For $f(x) = xsin\ x$, find the Fourier coefficient a_5.

(9) Define periodic function. Give an example of it.

(10) Find the pole and its order of the function $f(z) = \frac{\sin z}{z^4}$.

(11) Identify the type of singularities of the function $f(z) = e^{\frac{1}{z-1}}$.

(12) Define: Conformal mappings. At which points is the mapping $w = z^2 + \frac{1}{z^2}$ not conformal?

**** ****