2004N352

Candidate's Seat No :________

M.Sc Semester-2 Examination

MAT410

Partial Differential Equations

April 2024

Time: 2-30 hours] **[Max. Marks:70**

Instructions: All questions are compulsory. Use of non-programmable scientific calculator is allowed.

Q.1 **(a)** Find the general integral of the equation $xp - yq = \frac{y^2-x^2}{z}$ **(07)**

(b) Check compatibility of given equations and find their solution: **(07)**

$$xp = yq\ , z(xp + yq) = 2xy$$

OR

Q.1 **(a)** If $\overrightarrow{X} \cdot curl\overrightarrow{X} = 0$ where $\overrightarrow{X} = (P, Q, R)$ and μ is an arbitrary differentiable function of x, y and z, then show that $\mu\overrightarrow{X} \cdot curl(\mu\overrightarrow{X}) = 0$. **(07)**

(b) Find the complete integral of the equation $2zx - px^2 - 2qxy + pq = 0$. **(07)**

Q.2 **(a)** Find a complete integral of the equation $p^2x + q^2y = z$ by Jacobi's method. **(07)**

(b) Find a complete integral of the equation **(07)**

$$z = p^2 - q^2$$

and the integral surface passing through the curve with the equation $4z + x^2 = 0, y = 0$.

OR

Q.2 **(a)** Find the integral surface of the equation **(07)**

$$2y(z-3)p + (2x - z)q = y(2x - 3)$$

which pass through the circle $z = 0,\ x^2 + y^2 = 2x$.

(b) Find the characteristics of the equation $pq = z$ and hence, determine the integral surface which pass through the parabola $x = 0, y^2 = z$. **(07)**

Q.3 **(a)** Reduce the equation $\frac{\partial^2 z}{\partial x^2} = x^2 \frac{\partial^2 z}{\partial y^2}$ to canonical form. **(07)**

(b) State the problem of the wave equation in the case of vibrations of a string of finite length and solve it using the method of separation of variables. Consider that both ends are fixed and initial displacement distribution is $f(x)$ and initial velocity distribution is $g(x)$. **(07)**

OR

Q.3 **(a)** Solve the initial and boundary value problem: **(07)**

$y_{tt} - 4y_{xx} = 0, \quad 0 < x < 1, \quad t > 0$

IC: $y(x, 0) = x(1 - x)$

$y_t(x, 0) = 0$

BC: $y(0, t) = 0$

$y(1, t) = 0$

(b) State and solve the heat conduction problem for an infinite rod case with initial temperature distribution in the rod at time $t = 0$ given by $f(x)$. Use Fourier Transform method. **(07)**

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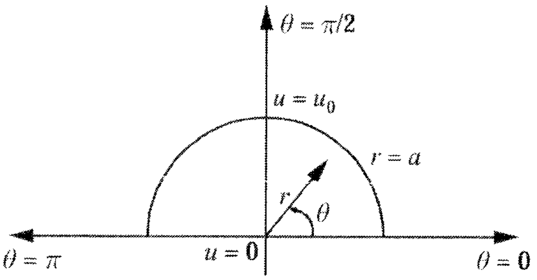
Q.4 **(a)** State and solve the Dirichlet problem for a rectangle. **(07)**

(b) State and solve the Neumann problem for the upper half plane. **(07)**

OR

Q.4 **(a)** State and solve the Dirichlet problem for the interior of the circle. **(07)**

(b) Find the steady state temperature distribution in a semi-circular plate of radius a, insulated on both the faces with its curved boundary kept at a constant temperature U_0 and its bounding diameter kept at zero temperature as described in Fig. **(07)**



Q.5 Attempt any **SEVEN** out of **TWELVE:** **(14)**

(1) Eliminate the arbitrary function from the following equation and hence, obtain the corresponding partial differential equation

$$z = f(x^2 - y^2)$$

(2) Find the complete integral of the equation $x(1+y)p = y(1+x)q$

(3) Find the general integral of the equation $p + q = z$

(4) When will you say that the equations are said to be compatible?

(5) Find the Jacobian $\frac{\partial(u,v)}{\partial(x,y)}$ for the function $u = e^x \sin y, v = (x + \log \sin y)$

(6) State Green's identities

(7) Find the characteristic of the equation $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = 0$

(8) State (only) Harnack's theorem

(9) Show that $\sinh x \cos y$ satisfying the Laplace equation when x and y are Cartesian coordinates.

(10) Prove that the solution of the Dirichlet problem, if it exists, is unique.

(11) Define: Dirichlet boundary value problem

(12) What is the necessary condition for the existence of the solution U of the problem $\nabla^2 U = 0$ in a bounded domain D, and $\frac{\partial U}{\partial n} = f(x)$ on the boundary B, where $\frac{\partial}{\partial n}$ is the directional derivative along the outward normal?

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