1904N298 Candidate's Seat No :

MSc-SEM-II (April 2024)
MATHEMATICS
MAT 409 (Complex Analysis II)

Time: 2 ½ hours M.M. 70

Instructions:

1. All questions in Section I carry equal marks.
2. Attempt any **7** questions from Section II.

Section I

Q.1 A. State and prove Taylor's Theorem. 7

B. Show that $\frac{1}{1-z}=\sum_{n=0}^{\infty} z^n \; (|z|<1)$, and by differentiating the Maclaurin Series Representation obtain the expansion $\frac{1}{(1-z)^2}=\sum_{n=0}^{\infty}(n+1)z^n \; (|z|<1)$. 7

OR

A. State and prove Laurent's Theorem. 7

B. Give two Laurent series expansion in power of z for the function $f(z)=\frac{1}{z^2(1-z)}$ and specify the region in which those expansions are valid. 7

Q.2 A. Let $p(z)$ and $q(z)$ be two polynomials such that $\deg(q(z)) \geq \deg(p(z)) + 2$ and assume that C is a simple closed contour such that interior of C contains all the zeroes of $q(z)$. Then, show that 7

$$\int_C \frac{p(z)}{q(z)}\,dz = 0.$$

B. State and prove Riemann's removable singularity theorem. 7

OR

A. Use Cauchy-residue theorem to evaluate the following integral 7

$$\int_{|z|=4} \frac{3z^2+2}{(z-1)(z^2+9)}\,dz.$$

B. Let f be an analytic function at z_0 then prove that z_0 is a zero of order m of f if and only if there is an analytic function g, which is non-zero at z_0, such that 7

$$f(z) = (z - z_0)^m g(z).$$

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Q.3 A. Evaluate the improper integral $\int_0^{\infty} \frac{x \sin 2x}{x^2+3} dx$. 7

B. Derive the integration formula by integrating $\int_0^{\infty} \frac{x^{-a}}{x+1} dx$. Use 7

$f(z) = \frac{z^{\frac{-1}{2}}}{z^2+1} for\ |z| > 0, 0 < arg\ z < 2\pi$.

OR

A. Prove the integral $\int_0^{\infty} \frac{\cos ax - \cos bx}{x^2} dx = \frac{\pi}{2}(b-a)$. 7

B. Find the integral $\int_0^{2\pi} \frac{d\theta}{1+a \sin\theta} = \frac{2\pi}{\sqrt{1-a^2}}$ where $-1 < a < 1$. 7

Q.4 A. State and prove Rouche's Theorem and Determine the number of zeros with counting multiplicities of the polynomial $p(z) = z^6 - 5z^4 + z^3 - 2z$ inside the contour $|z| = 1$. 7

B. If $f(z) = \frac{\left(z^7+1\right)^5 \left(z - \frac{1}{9}\right)^6}{\left(z^2 + \frac{1}{7}\right)^{11} (z-2i)^3}$ then find $\frac{1}{2\pi i} \oint_{|z|=5} \frac{f'(z)}{f(z)} dz$. 7

OR

A. Find the image of the infinite strip $\frac{1}{4} \le y \le \frac{1}{2}$ under transformation $w = \frac{1}{z}$. 7

B. $f(z)$ be a Mobius Transformation such that $f(\infty) = 1, f(i) = i, f(-i) = -i$ find the image of the unit disk $\{z \in \mathbb{C} : |z| < 1\}$ under $f(z)$. 7

Section II

Q.5 **Attempt any 7** 14

1. The sequence $z_n = \frac{1}{n^3} + i \ (n = 1, 2, ...)$ converges to

a. i

b. $-i$

c. 0

d. 1

2. The seris expansion of $\sin z$ is

a. $\sum_{n=0}^{\infty} (-1)^n \frac{z^{2n+1}}{(2n+1)!}$

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b. $\sum_{n=0}^{\infty}(-1)^n \frac{z^{2n+2}}{(2n+1)!}$

c. $\sum_{n=0}^{\infty}(-1)^n \frac{z^{2n+2}}{(2n)!}$

d. $\sum_{n=0}^{\infty} \frac{z^{2n+1}}{(2n+1)!}$

3. The Third term in the Taylor series expansion of e^z is

a. $\frac{z^3}{2}$

b. 1

c. $\frac{z^2}{2}$

d. None of these

4. The residue of the function $\frac{exp(-z)}{(z-1)^2}$ at $z = 1$ is

a. e b $1/e$ c. $-e$ d. $-1/e$

5. Which of the following two statements is/are always true?

I. Suppose $f(z)$ is analytic on and inside a simple closed curve C and $f(z)$ has n zeroes inside C then $f'(z)$ has $n - 1$ zeroes inside C.

II. Suppose $f(z)$ has a pole of order m at $z = 0$, then $f(z^2)$ has pole of order $2m$ at $z = 0$.

a. Both are true.

b. I is true and II is false.

c. I is false and II is true.

d. Both are false.

6. The residue of the function $f(z) = \frac{z-1}{z+1}$ at $z = \infty$ is

a. 2

b. -2

c. 0

d. $-1/2$

7. Which of the following is a multivalued function?

a. $z + 1$

b. z

c. $log\, z$

d. $z - 1$

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8. Which of the following is the branch point of $z^{1/2}$?

a. 0

b. 1 and -1

c. $\pi + 1$

d. None of the above

9. The value of $Log(-ei)$ is _____.

a. $-\frac{\pi}{2}i$

b. $1+\frac{\pi}{2}i$

c. $1-\frac{\pi}{2}$

d. $1-\frac{\pi}{2}i$

10. Find the number of zeros with counting multiplicities of the polynomial $z^4-2z^3+9z^2+z-1$ inside the circle $C:|z|=2$.

a. 4 b. 3 c. 2 d. 1

11. . Consider the following lines $L_1=\{z\in\mathbb{C}\,|\,2x+3y=0\}$ and $L_2=\{z\in\mathbb{C}\,|\,\mathrm{Re}(z)=1\}$. The image of L_1 and L_2 respectively under the inversion mapping $w=\frac{1}{z}$ is a

a. Line and Circle b Line c.Circle d. Circle and Line

12. The fixed point of translation mapping is $w=\frac{2z+3}{5z-1}$

a. 0 b. 1 c. 2 d. 3