1904M119 Candidate's Seat No :________

Integ. M.Sc AIML Semester-4 Examination

CC-213

Vector Calculus

April-2024

Time : 2-30 Hours] **[Max. Marks : 70**

Instructions: All questions are compulsory. Use of non-programmable scientific calculator is allowed.

Q.1 **(a)** Define: unit tangent, unit normal and unit binormal vectors for the curve and find the unit tangent, unit normal and unit binormal vectors for the curve: **(07)**
$x = t, y = 3sint, z = 3cost.$

(b) State (only) Curvature and Torsion of a space curve. Find curvature and torsion of a curve $\gamma(t) = 3cost\hat{\imath} + 3sint\hat{\jmath} + 2t\hat{k}.$ **(07)**

OR

(a) Show that the vector-valued function: **(07)**
$\gamma(t) = (2i + 2j + k) + cost\left(\frac{1}{\sqrt{2}}i - \frac{1}{\sqrt{2}}j\right) + sint\left(\frac{1}{\sqrt{3}}i + \frac{1}{\sqrt{3}}j + \frac{1}{\sqrt{3}}k\right)$ describes the motion of a particle moving in the circle of radius 1 centered at the point $(2,2,1)$ and lying in the plane $x + y - 2z = 2$.

(b) A particle moves along a plane curve such that its linear velocity is perpendicular to the radius vector. Show that the path of the particle is a circle. **(07)**

Q.2 **(a)** The temperature of the points in space is given by $\varphi = x^2 + y^2 - z$. A mosquito located at point (1,1,2) desires to fly in such a direction that it will get warm as soon as possible. In what direction should it move? **(07)**

(b) Find the directional derivative of the function $\varphi = x^2y + y^2z + z^2x^2$ at $(1,2,1)$ in the direction of the normal to the surface $x^2 + y^2 - z^2x = 1$ at $(1,1,1)$. **(07)**

OR

(a) Find the directional derivative of the function $\varphi = xy^2 + yz^2 + zx^2$ along the tangent to the curve $x = t, y = t^2, z = t^3$ at the point $(1,1,1)$. **(07)**

(b) Find the angle of intersection of the spheres $x^2 + y^2 + z^2 = 29$ and $x^2 + y^2 + z^2 + 4x - 6y - 8z = 47$ at $(4, -3, 2)$. **(07)**

Q.3 **(a)** Show that the vector field $\bar{A} = \frac{a(x\hat{\imath}+y\hat{\jmath})}{\sqrt{x^2+y^2}}$ is a source field or sink field according as $a > 0$ or $a < 0$. **(07)**

(b) Find the area of the region bounded by the parabola $y = x^2$ and the line $y = x + 2$. **(07)**

OR

(P.T.O)

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(a) Evaluate $\int_C \bar{F} . d\bar{r}$ where $\bar{F} = (x^2 + y^2)\hat{\imath} - 2xy\hat{\jmath}$ and C is the rectangle in the xy –plane bounded by $y = 0, x = a, y = b, x = 0$. **(07)**

(b) Find the work done in moving a particle in the force field: $\bar{F} = 3x^2\hat{\imath} + (2xz - y)\hat{\jmath} + z\hat{k}$ along the curve $x^2 = 4y$ and $3x^3 = 8z$ from $x = 0$ to $x = 2$. **(07)**

Q.4 (a) From the following premises, by giving appropriate reasons prove that 't' is a conclusion. **(07)**

1. $\sim p \wedge q$
2. $r \rightarrow p$
3. $\sim r \rightarrow s$
4. $s \rightarrow t$

(b) Suppose we remove a square from a standard 8×8 chessboard then by Principle of Mathematical Induction show that one can tile the 63 remaining squares of chessboard by L-shaped triominoes. **(07)**

OR

(a) State (only) the Pigeonhole Principle. Show that if any 11 numbers are chosen from the set $\{1,2, \ldots ,20\}$, then one of them will be a multiple of another. **(07)**

(b) Find explicit form of the recurrence relation $a_n = 3a_{n-1} - 2a_{n-2}$ with initial conditions $a_1 = 5$ and $a_2 = 3$. **(07)**

Q.5 Attempt any **SEVEN** out of **TWELVE:** **(14)**

(1) Find direction in which temperature changes most rapidly with distance from point (1,1,1) and determine maximum rate of change, if temperature at any point is given by $\emptyset = xy + yz + zx$.

(2) Find the arc length of a particle along a curve $r(t) = cost\hat{\imath} + sint\hat{\jmath} + t\hat{k}$ from $t = 0$ to $t = 2\pi$.

(3) How many friends must you have to guarantee that atleast five of them will have birthdays in the same month?

(4) State (only) Green's Theorem in the plane and interpret its equation equality.

(5) Define radius of curvature. What is the radius of curvature of a circle $x^2 + y^2 = 2$.

(6) Explain in brief the physical interpretation of Divergence of a vector field.

(7) For vector field $\bar{F} = x^2y\hat{\imath} - 2xz\hat{\jmath} + 2yz\hat{k}$ find the curl at the point (1,0,2).

(8) Determine the constant a if vector $\bar{F} = (x - 5z)\hat{\imath} + (2y - 3x^2)\hat{\jmath} + (3x^2 + az)\hat{k}$ is solenoidal.

(9) In how many ways can a prize winner choose three CDs from the Top Ten list if repeats are allowed?

(10) Explain whether the $g_n = g_{n-1}^2 - 4g_{n-2}$ is a linear homogeneous recurrence relation or not.

(11) Thirty cars were assembled in a factory. The options available were a radio, an air conditioner and white wall tires. It is known that 15 of the cars have radios, 8 of them have air conditioners and 6 of them have white-wall tires. Moreover, 3 of them have all three options. Determine at least how many cars do not have any options at all.

(12) Show that if seven colors are used to paint 50 bicycles, at least 8 bicycles will be the same color.

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