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1804N244

Candidate's Seat No :_____

M.Sc Semester-2 Examination
M.Sc. (Sem-II) Examination
AMS 408
Advanced Operations Research
April 2024

Time: 2-30 hours] [Max. Marks: 70

Instructions: All questions are compulsory. Use of non-programmable scientific calculator is allowed.

Q.1 (a) A food product company is contemplating the introduction of a revolutionary new product with new packaging to replace the existing product at much higher price (S_1), or a moderate change in the composition of the existing product with a new packaging at a small increase in price (S_2), or a small change in the composition of the existing except the word 'New' with a negligible increase in price (S_3). The three possible states of nature or events are: (i) high increase in sales (N_1), (ii) no change in sales (N_2), and (iii) decrease in sales (N_3). The marketing department of the company worked out the pay offs in terms of yearly net profits for each of the strategies of three events (expected sales). This is represented in the following Table. (07)

		States of Nature		
		N_1	N_2	N_3
Strategies	S_1	700000	300000	150000
	S_2	500000	450000	0
	S_3	300000	300000	300000

Which strategy should the concerned executive choose based on (a) Maximax Criterion (b) Maximin Criterion (c) Hurwicz criterion (farmer's degree of optimism being 0.6) (d) Laplace criterion (e) Minimax regret criterion?

(b) Two television stations compete with each other for viewing audience. Local programming options for the 5:00 P.M. weekday time slot include a sitcom rerun, an early news program, or a home improvement show. Each station has the same programming options and must make its preseason program selection before knowing what the other television station will do. The viewing audience gains in thousands of viewers for Station A are shown in the payoff table. (07)

		Situation B		
		Sitcom Rerun B_1	**News Program B_2**	**Home Improvement B_3**
Situation A	**Sitcom Rerun A_1**	10	-5	3
	News Program A_2	8	7	6
	Home Improvement A_3	4	8	7

a. Determine the optimal strategy for each station using LP method.
b. What is the value of the game?

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OR

(a) The Gorman Manufacturing Company must decide whether to manufacture a component part at its Milan, Michigan, plant or purchase the component part from a supplier. The resulting profit is dependent upon the demand for the product. The following payoff table shows the projected profit (in thousands of dollars): **(07)**

	State of Nature		
Decision Alternative	**Low Demand** S_1	**Medium Demand** S_2	**High Demand** S_3
Manufacture d_1	-20	40	100
Purchase d_2	10	45	70

The state-of-nature probabilities are $P(S_1) = 0.35$, $P(S_2) = 0.35$, and $P(S_3) = 0.30$.

a. Use a decision tree to recommend a decision.
b. Use EVPI to determine whether Gorman should attempt to obtain a better estimate of demand.
c. A test market study of the potential demand for the product is expected to report either a favorable (F) or unfavorable (U) condition. The relevant conditional probabilities are as follows:

$P(F|S_1) = 0.10$ $\quad$ $P(U|S_1) = 0.90$
$P(F|S_2) = 0.40$ $\quad$ $P(U|S_2) = 0.60$
$P(F|S_3) = 0.60$ $\quad$ $P(U|S_3) = 0.40$

d. What is Gorman's optimal decision strategy?
e. What is the expected value of the market research information?

(b) The offensive coordinator for the Chicago Bears professional football team is preparing a game plan for the upcoming game against the Green Bay Packers. A review of game tapes from previous Bears–Packers games provides data on the yardage gained for run plays and pass plays. Data shows that when the Bears run against the Packers' run defense, the Bears gain an average of 2 yards. However, when the Bears run against the Packers' pass defense, the Bears gain an average of 6 yards. A similar analysis of pass plays reveals that if the Bears pass against the Packers' run defense, the Bears gain an average of 11 yards. However, if the Bears pass against the Packers' pass defense, the Bears average a loss of 1 yard. This loss, or negative gain of –1, includes the lost yardage due to quarterback sacks and interceptions. Develop a payoff table that shows the Bears' average yardage gain for each combination of the Bears' offensive strategy to run or pass and the Packers' strategy of using a run defense or a pass defense. What is the optimal strategy for the Chicago Bears during the upcoming game against the Green Bay Packers? What is the expected value of this strategy? **(07)**

Q.2 (a) Solve following NLP by using separable programming method: **(07)**
$\max f(x) = x_1 + {x_2}^4; 3x_1 + 2{x_2}^2 \leq 9$

(b) A total of 160 hours of labor are available each week at \$15/hour. Additional labor can be purchased at \$25/hour. Capital can be purchased in unlimited quantities at a cost of \$5/unit of capital. If K units of capital and L units of labor are available during a week, then LK machines can be produced. Each machine sells for \$270. How can the firm maximize its weekly profits? **(07)**

OR

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(a) A beer company has divided Bloomington into two territories. If x_1 dollars are spent on promotion in territory 1, then $6(x_1)^{1/2}$ cases of beer can be sold there; and if x_2 dollars are spent on promotion in territory 2, then $4(x_2)^{1/2}$ cases of beer can be sold there. Each case of beer sold in territory 1 sells for \$10 and incurs \$5 in shipping and production costs. Each case of beer sold in territory 2 sells for \$9 and incurs \$4 in shipping and production costs. A total of \$100 is available for promotion. How can the beer company maximize profits? If an extra dollar could be spent on promotion, by approximately how much would profits increase? By how much would revenues increase? (07)

(b) Solve following NLP by using Lagrange Multiplier Method: (07)
$z = -{x_1}^2 - {x_2}^2 + 6x_1 + 8x_2$
Subject to the constraints: $4x_1 + 3x_2 = 16\,; 3x_1 + 5x_2 = 15$

Q.3 **(a)** DJS Investment Services must develop an investment portfolio for a new client. As an initial investment strategy, the new client would like to restrict the portfolio to a mix of two stocks: (07)

Stock	Price/Share	Estimated Annual Return/Share	Risk Index/share
AGA Products	\$ 25	\$ 3	0.50
Key Oil	\$ 50	\$ 5	0.25

The client wants to invest \$ 80,000 and established the following two investment goals:
Priority Level 1 Goal
Goal 1: the client agreed that an acceptable level of risk would correspond to portfolios with a maximum total risk index of 700.

Priority Level 2 Goal
Goal 2: Obtain an annual return of at least 9000.
Formulate a goal programming model and solve it by graphical method.

(b) $Min\ Z = p_1{d_1}^- + p_2(2{d_2}^- + {d_3}^-) + p_3{d_1}^+$ (07)
Subject to the constraints: $x_1 + x_2 + {d_1}^- - {d_1}^+ = 400;$
$x_1 + {d_2}^- = 240;$
$x_2 + {d_3}^- = 300$

OR

(a) Harrison Electric produces two products popular with home renovators, old fashioned chandeliers and ceiling fans. Both chandeliers and fans require a two-step production process involving wiring and assembly. It takes about 2 hrs to wire a chandelier and 3 hrs to wire a fan. Final assembly of the chandelier and fan requires 6 and 5 hrs respectively. The production capability is such that only 12 hrs of wiring and 30 hrs of assembly time are available. Each chandelier produced nets the firm \$7 and each fan \$6. The Harrison's management wants to achieve the following goals with the given priorities: (07)

*P*1: Reach a profit as much above \$30 as possible.
*P*2: Fully use wiring department hours available.
*P*3: Avoid assembly department overtime.
*P*4: Produce at-least 7 ceiling fans.

Formulate the above goal programming problem and solve it graphically.

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(b) Industrial Chemicals produces two adhesives used in the manufacturing process for airplanes. The two adhesives, which have different bonding strengths, require different amounts of production time: the IC-100 adhesive requires 20 minutes of production time per gallon of finished product, and the IC-200 adhesive uses 30 minutes of production time per gallon. Both products use 1 pound of a highly perishable resin for each gallon of finished product. Inventory currently holds 300 pounds of resin, and more can be obtained if necessary. However, because of the limited shelf life of the material, any amount not used in the next two weeks will be discarded. **(07)**

The firm has existing orders for 100 gallons of IC-100 and 120 gallons of IC-200. Under normal conditions, the production process operates eight hours per day, five days per week. Management wants to schedule production for the next two weeks to achieve the following goals:

Priority Level 1 Goals

Goal 1: Avoid underutilisation of the production process.

Goal 2: Avoid overtime in excess of 20 hours for the two weeks.

Priority Level 2 Goals

Goal 3: Satisfy existing orders for the IC-100 adhesive; that is, produce at least 100 gallons of IC-100.

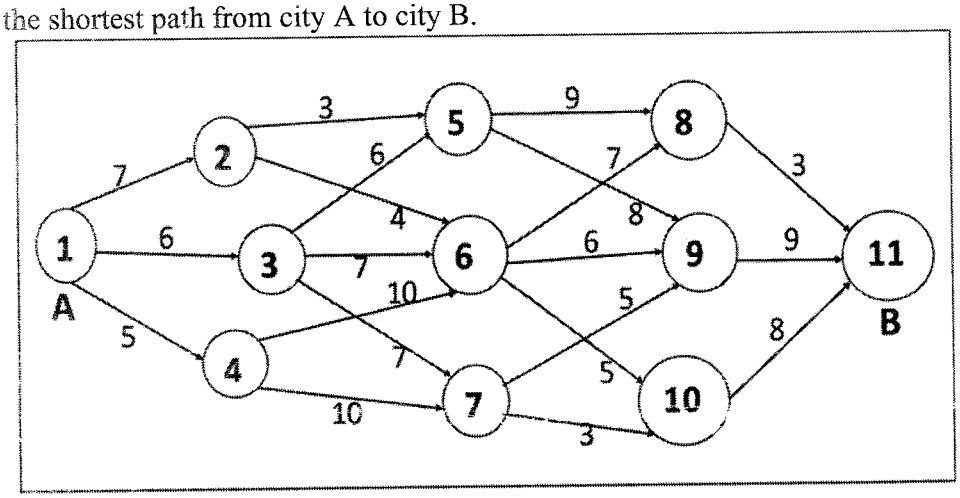
Goal 4: Satisfy existing orders for the IC-200 adhesive; that is, at least a20 gallons of IC-200.

Priority Level 3 Goals

Goal 5: Use all the available resin.

Formulate a goal programming model for the Industrial Chemicals problem. Assume that both priority level 1 goals and both priority level 2 goals are equally important.

Q.4 (a) Find the shortest path from city A to city B. **(07)**



(b) A Construction contractor estimates that the size of the work force needed over the next 5 weeks to be 5, 7, 8, 4 and 6 workers, respectively. Excess labor kept on the force will cost $300 per worker per week and new hiring in any week will incur a fixed cost of $400 plus $200 per worker per week. Find the number of labors per week such that the total cost of labor needed for the project is minimized. **(07)**

OR

(a) A 6-ton vessel is loaded with one or more of the three items. The following table gives the unit weight w_i in tons and the unit revenue in thousands of rupees r_i, for item i. How should the vessel be loaded to maximize the total return? (07)

Item i	w_i	r_i
1	4	70
2	1	20
3	2	40

(b) A company needs to determine the optimal replacement policy for a current 2-year-old machine over the next 4 years. The company requires that a 6-year-old machine be replaced. The cost of a new machine is \$100,000. The following table gives the data of the problem. (07)

Age, t (year)	Revenue, $r(t)$ (\$)	Operating cost, $c(t)$ (\$)	Salvage value, $s(t)$ (\$)
0	20,000	200	-
1	19,000	600	80,000
2	18,500	1200	60,000
3	17,200	1500	50,000
4	15,500	1700	30,000
5	14,000	1800	10,000
6	12,200	2200	5000

Q.5 Attempt any **SEVEN** out of **TWELVE:** (14)

(1) Explain: Pure strategy and Mix strategy

(2) Farmer McCoy can plant either corn or soybeans. The probability that the next harvest prices will go up, stay the same, or go down are 0.25, 0.30, and 0.45, respectively. If the prices go up, the corn crop will net \$30,000 and the soybeans will net \$10,000. If the prices remain unchanged, McCoy will (barely) break even. But if the prices go down, the corn and soybeans crops will sustain losses of \$35,000 and \$5000, respectively. Represent McCoy's problem as a decision tree.

(3) Define efficiency value of sample information.

(4) The cost per day of running a hospital is $200{,}000 + 0.002x^2$ dollars, where x =patients served per day. What size hospital minimizes the per-patient cost of running the hospital?

(5) Check definiteness of function: $f(x) = {x_1}^2 + 2{x_2}^2 - 7{x_3}^2 - 4x_1x_2 + 8x_1x_3$

(6) Write down necessary and sufficient conditions for Kuhn Tucker Method.

(7) Define Preemptive and non-Preemptive Goal Programming Problem.

(8) Two products are manufactured on two sequential machines. The following table gives the machining times in minutes per unit for the two products.

	Machining time in min	
Machine	Product 1	Product 2
1	5	3
2	6	2

The daily production quotas for the two products are 80 and 60 units respectively. Each machine runs 8 hours a day. Overtime, though not desirable, may be used if necessary to meet the production quota. Formulate the problem as a goal programming model.

(9) Explain Deviation variable in goal programming problem.
(10) Write down the difference between linear programming and dynamic programming.
(11) Define stages and states.
(12) Define Recursive relationship in dynamic programming.

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