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# 1804N243

Candidate's Seat No :________

**M.Sc Semester-2 Examination**
**April -2024**
**Mathematics**
**MAT408 Real Analysis**

**Time: 2.30 Hours** **Marks: 70**

1. (A) State and prove Luzin's theorem. **7**

   (B) Find the Bernstein polynomial $B_3(x)$ for $f(x) = \sin(\pi x)$ defined on $[0, 1]$. **7**

**OR**

1. (A) Show that there is no polynomial $p(x)$ defined on $E = [-1, 1]$ such that $mE(p(x) \neq |x|) < \frac{1}{3}$. **7**

   (B) Define the convergence in measure. If a sequence $(f_n)$ converges in measure to two functions $f$ and $g$ on $E$ then prove that $f = g$ a.e.on $E$. **7**

2. (A) Prove that $C[a, b]$ is a dense subset of the space $(L_2[a, b], ||\cdot||_2)$. **7**

   (B) By an example, show that if $f_n(x) \to f(x)$ a.e. then it is not necessary that $f_n(x) \to f(x)$ in the mean of order $p$. **7**

**OR**

2. (A) Give a sequence $(f_n)$ such $f_n(x) \to f(x)$ in the mean of order $p$ but $(f_n)$ does not converge to $f$ pointwise everywhere. Justify. **7**

   (B) Prove or disprove that $C[0, 1]$ is a closed linear subspace of space $(L_p[0, 1], ||\cdot||_p)$. **7**

3. (A) Let $f : [a, b] \to \mathbb{R}$ be increasing. Then prove that $f'$ is measurable on $[a, b]$ and $\int_a^b f'(x)dx \leq f(b) - f(a)$. **7**

   (B) Give an example of a uniformly continuous function on $[0, 1]$ that is not absolutely continuous on $[0, 1]$. Explain. **7**

**OR**

3. (A) If $f : [a, b] \to \mathbb{R}$ is of finite variation then prove that $f$ can be expressed as a sum of two monotone functions. **7**

   (B) Give an example of a continuous function on $[0, 1]$ that is not of finite variation. Explain. **7**

4. (A) Prove that $f \in C^1[a, b]$ if and only if $f$ is an indefinite integral of some continuous function on $[a, b]$ ( in the sense of Riemann). **7**

   (B) Find the Fourier series for function $f$ defined on $[-\pi, \pi]$ by $f(x) = x$. **7**



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**OR**

4. (A) Find the Fourier series for function $f$ defined on $[-\pi, \pi]$ by $f(x) = |x|$. **7**

   (B) Give an example of $f \in C^2[0,1]$ such that $f \notin C^3[0,1]$. Explain **7**

5. **Attempt any SEVEN of the following:** 14

   (1) Consider $f : [0,1] \to \mathbb{R}$ defined by $f(x) = \sqrt{x}$. Then which of the following statements are true?

   (A) $f$ is continuous
   (B) $f$ is of finite variation
   (C) $f$ is absolutely continuous
   (D) $f$ is Lipschitz continuous

   (2) What is the least value of $k > 0$ such that $|\sin^2 x - \sin^2 y| \leq k|x - y|$?

   (A) $\frac{1}{2}$ (B) 1 (C) $\frac{3}{2}$ (D) 2

   (3) The $L_2$ norm of $f(x) = \sin x$ ( defined on $[-\pi, \pi]$) is ________

   (A) 0 (B) 1 (C) $\sqrt{\pi}$ (D) $\sqrt{2\pi}$

   (4) What is the conjugate index of $p = 6$?

   (A) $\frac{1}{6}$
   (B) $\frac{6}{5}$
   (C) $\frac{5}{6}$
   (D) $\frac{-1}{6}$

   (5) Find $||x||_2$, where $x = (1, \frac{1}{2}, \frac{1}{4}, \frac{1}{16}, ...) \in l_2$.

   (A) 1 (B) $\frac{4}{3}$ (C) $\sqrt{\frac{4}{3}}$ (D) none of these

   (6) The sequence $f_n(x) = \frac{x^n}{1+x^n}$ over the interval $[0,2]$ ________

   (A) converges uniformly
   (B) does not converge uniformly

   (7) Let $f : [0,1] \to \mathbb{R}$ be defined by $f(x) = \sin \frac{1}{x}$, when $x \neq 0$ and $f(0) = 1$.
   What is the total variation of $f$ on $[0,1]$?

   (A) 0
   (B) 1
   (C) $2\pi$
   (D) none of the above

   (8) Suppose $f(x)$ is an even function in $L_1[-\pi, \pi]$. Then it follows that the Fourier coefficients ________

   (A) $a_k = 0$ for all $k \geq 0$
   (B) $a_k + b_k = 0$ for all $k \geq 1$
   (C) $b_k = 0$ for all $k \geq 1$
   (D) $a_k - b_k = 0$ for all $k \geq 1$

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(9) Which of the following statements are true for the Cantor function $\theta$ on $[0, 1]$?

(A) $\theta$ is uniformly continuous

(B) $\theta$ is Lipschitz continuous

(C) $\theta$ is of finite variation

(D) $\theta$ is absolutely continuous

(10) Which of the following sequences belong to the space $l_2$?

(A) $(1, 0, 1, 0, 1, 0, 1, 0, ...)$

(B) $(\log 1, \log 2, \log 3, \log 4...)$

(C) $(0, 0, 0, 0, 0, 0, 0, ...)$

(D) $(1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, ...)$

(11) Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function. Which of the following imply that it is uniformly continuous?

(A) $f$ is $2\pi$- periodic

(B) $f$ is differentiable and $f'$ is bounded

(C) $f$ is absolutely continuous

(D) none of the above

(12) The derivative of the Cantor $\theta$ is equal to zero on a set of measure ________

(A) 0

(B) 1

(C) $\frac{1}{2}$

(D) 2