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1804N218

Candidate's Seat No :________

M.Sc Semester-2 Examination

408

Statistics

Time : 2-30 Hours] **April-2024** **[Max. Marks : 70**

Note: Attempt all questions.

Q.1

(i) Define Neyman type-A distribution. Derive its probability mass function (pmf) using probability generating function(pgf). Obtain mean and variance of the distribution. [7]

(ii) Show that Neyman type-B distribution tends to Neyman type-A distribution if [7]
(i) $np = \theta$ (fixed) (ii) $n \to \infty$.

OR

(i) Define Neyman type-B distribution. Derive its probability mass function (pmf) using probability generating function(pgf). Obtain mean and variance of the distribution. [7]
(ii) Define Poisson-Negative binomial distribution. Obtain its pgf and recurrence relation for the probability. [7]

Q.2

(i) Define non-central chi-square distribution. Obtain its moment generating function (mgf). [7]

(ii) State and prove additive or re-productive property of non-central chi-square distribution. [7]

OR

(i) Discuss cumulants of non-central chi-square distribution. [7]
(ii) Define non-central 'F' distribution. Obtain its mean and variance. [7]

Q.3

(i) For the exponential distribution $f(x) = e^{-x}, x \geq 0$, show that the cumulative distribution function (c.d.f.) of $X_{(n)}$ in a random sample of size n is $F_n(x) = (1 - e^{-x})^n$. Hence prove that as $n \to \infty$, the c.d.f. of $X_{(n)} - \log n$ tends to the limiting form $\exp[-e^{-x}]$, $-\infty < x < \infty$. [7]

(ii) Show that for a random sample of size 2 from $N(0, \sigma^2)$ population, $E(X_{(1)}) = -\sigma / \sqrt{\pi}$. [7]

OR

(i)Show that in odd samples of size n from $U[0, 1]$ population, the mean and variance of the distribution of median are $1/2$ and $1/[4(n+2)]$ respectively. [7]

(ii) Find the p.d.f. of $X_{(r)}$ in a random sample of size n from the exponential distribution: [7]

$$f(x) = \alpha e^{-\alpha x}, \alpha > 0, x \geq 0$$

Also show that $X_{(r)}$ and $W_{rs} = X_{(s)} - X_{(r)}$, $r < s$, are independently distributed.

P.T.O

Q.4 NV218 – 2

(i) Explain Wilcoxon signed rank test with example. [7]

(ii) Explain correlation between variate values and ranks with example. [7]

OR

(i) Explain Wilcoxon rank-sum test for two independent samples with example. [7]

(ii)Describe rank order statistics with appropriate example. [7]

Q.5 Answer any seven: [14]

(i) Give one application of contagious distribution.

(ii) For Poisson-Binomial distribution mean is greater than variance.

(a)True (b) False

(iii) Neyman type- A distribution tends to Normal distribution when

(a) $\lambda \to \infty$ (b) $m \to \infty$ (c) $\lambda \to \infty$ and $m \to \infty$ (d) $\lambda \to \infty$ and $m \to 0$

(iv) Give one application of non-central t distribution.

(v) Give one application of non-central chi-square distribution.

(vi) Give one application of non-central F distribution.

(vii) In usual notations, the c.d.f. of the largest order statistic $X_{(n)}$ is given by

(a) $F_n(x) = [F(x)]^n$ (b) $F_n(x) = [F(x)]^{n+1}$ (c) $F_n(x) = [1 - F(x)]^n$ (d) $F_n(x) = [1 + F(x)]^n$

(viii) In usual notations, the c.d.f. of the smallest order statistic $X_{(1)}$ is given by

(a) $F_1(x) = 1 - [1 - F(x)]^n$ (b) $F_1(x) = 1 + [1 - F(x)]^n$

(c) $F_1(x) = 1 - [1 - F(x)]^{n+1}$ (d) none of the above

(ix) Wilcoxon signed rank test is a parametric test.

(a)True (b) False

(x) The sign test is a non-parametric alternative to the one-sample ______ test.

(xi) Give a situation where you prefer sign test in place of Wilcoxon signed rank test.

(xii) What do you understand by distribution of range?

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