7/46

# 1804M070

Candidate's Seat No :_____

**Integ. M.Sc DS Semester-4 Examination**

**CC-212**

**Statistical Inference (T)**

**Time : 2-30 Hours]** **April-2024** **[Max. Marks : 70**

**Instructions:** All questions are compulsory. Use of non-programmable scientific calculator is allowed.

**Q.1** **(a)** The National Quality Research Center at the University of Michigan provides a quarterly measure of consumer opinions about products and services. A survey of 10 restaurants in the Fast Food/Pizza group showed a sample mean customer satisfaction index of 71. Past data indicate that the population standard deviation of the index has been relatively stable with $\sigma = 5$. **(07)**

- **a.** Using 95% confidence, what is the margin of error?
- **b.** What is the margin of error of 99% confidence is desired?
- **c.** Develop a 90% confidence interval.
- **d.** Develop a 95% confidence interval.

**(b)** In an effort to estimate the mean amount spent per customer for dinner at a major Atlanta restaurant, data were collected for a sample of 49 customers. Assume a population standard deviation of Rs. 5. **(07)**

- **a.** At 95% confidence, what is the margin of error?
- **b.** If the sample mean is Rs. 24.80, what is the 95% confidence interval for the population mean?

**OR**

**(a)** In an automotive safety test conducted by the New Delhi Highway Safety Research Center, the average tire pressure in a sample of 62 tires was found to be 24 pounds per square inch, and the standard deviation was 2.1 pounds per square inch. **(07)**

- **a.** What is the estimated population standard deviation for this population? (There are about a million cars registered in New Delhi.)
- **b.** Calculate the estimated standard error of the mean.
- **c.** Construct a 95 percent confidence interval for the population mean.

**(b)** The standard deviation for a set of data is estimated to be 9. **(07)**

- **a.** At 95% confidence, how large a sample would provide a margin of error of 3?
- **b.** At 90% confidence, how large a sample would provide a margin of error of 2?

**Q.2** **(a)** In a study entitled How Undergraduate Students Use Credit Cards, it was reported that undergraduate students have a mean credit card balance of Rs. 3173. This figure was an all-time high and had increased 44% over the previous five years. Assume that a current study is being conducted to determine if it can be concluded that the mean credit card balance for undergraduate students has continued to increase compared to the April 2009 report. Based on previous studies, use a population standard deviation $\sigma = 1000$. **(07)**

- **a.** State the null and alternative hypotheses.
- **b.** What is the $p$-value for a sample of 180 undergraduate students with a sample mean credit card balance of Rs. 3325?
- **c.** Using $\alpha = 0.05$ level of significance, what is your conclusion?

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MAO -2

**(b)** A study found that, in 2005, 12.5% of U.S. workers belonged to unions. Suppose a sample of 400 U.S. workers is collected in 2006 to determine whether union efforts to organize have increased union membership. **(07)**

**a.** Formulate the hypotheses that can be used to determine whether union membership increased in 2006.

**b.** If the sample results show that 52 of the workers belonged to unions, what is the $p$-value for your hypothesis test?

**c.** At $\alpha = 0.05$, what is your conclusion?

**OR**

**(a)** A shareholders' group, in lodging a protest, claimed that the mean tenure for a chief executive office (CEO) was at least nine years. A survey of companies reported in The Wall Street Journal found a sample mean tenure of $\bar{x} = 7.27$ years for CEOs with a standard deviation of $s = 6.38$ years. **(07)**

**a.** Formulate hypotheses that can be used to challenge the validity of the claim made by the shareholders' group.

**b.** Assume 85 companies were included in the sample. What is the p-value for your hypothesis test?

**c.** At $\alpha = 0.01$, what is your conclusion?

**(b)** Fowle Marketing Research, Inc., bases charges to a client on the assumption that telephone surveys can be completed within 15 minutes or less. If more time is required, a premium rate is charged. With a sample of 35 surveys, a population standard deviation of 4 minutes, and a level of significance of .01, the sample mean will be used to test the null hypothesis $H_0: \mu \leq 15$. **(07)**

**a.** What is the probability of making a Type II error when the actual mean time is $\mu = 17$ minutes?

**b.** What is the probability of making a Type II error when the actual mean time is $\mu = 18$ minutes?

**Q.3** **(a)** Consider the following hypothesis test. **(07)**

$$H_0: \mu_1 - \mu_2 = 0$$
$$H_0: \mu_1 - \mu_2 \neq 0$$

| Sample 1 | Sample 2 |
|---|---|
| $n_1 = 80$ | $n_2 = 70$ |
| $\overline{x_1} = 104$ | $\overline{x_2} = 106$ |
| $\sigma_1 = 8.4$ | $\sigma_2 = 7.6$ |

**a.** What is the value of the test statistic?

**b.** Use $\alpha = 0.05$. What is the $p$-value and what is your conclusion?

**(b)** Are nursing salaries in Tampa, Florida, lower than those in Dallas, Texas? Salary data show staff nurses in Tampa earn less than staff nurses in Dallas. Suppose that in a follow-up study of 40 staff nurses in Tampa and 50 staff nurses in Dallas you obtain the following results. **(07)**

| Tampa | Dallas |
|---|---|
| $n_1 = 40$ | $n_2 = 50$ |
| $\bar{x}_1 = 56{,}100$ | $\overline{x_2} = 59{,}400$ |
| $s_1 = 6000$ | $s_2 = 7000$ |

M 70 – 3

Formulate hypothesis so that, if the null hypothesis is rejected, we can conclude that salaries for staff nurses in Tampa are significantly lower than for those in Dallas. Use $\alpha = 0.05$.

**a.** What is the value of the test statistic?

**b.** What is the $p$-value and what is your conclusion?

**OR**

**(a)** The following results come from two independent random samples taken of two populations. **(07)**

| **Sample 1** | **Sample 2** |
|---|---|
| $n_1 = 50$ | $n_2 = 35$ |
| $x_1 = 13.6$ | $x_2 = 11.6$ |
| $\sigma_1 = 2.2$ | $\sigma_2 = 3.0$ |

**a.** What is the point estimate of the difference between the two population means?

**b.** Provide a 90% confidence interval for the difference between the two population means.

**c.** Provide a 95% confidence interval for the difference between the two population means.

**(b)** A quality control supervisor for an automobile manufacturer is concerned with uniformity in the number of defects in case coming off the assembly line. If one assembly line has significantly more variability in the number of defects, then changes have to be made. The supervisor has collected the following data: **(07)**

| | **Number of Defects** | |
|---|---|---|
| | **Assembly line A** | **Assembly line B** |
| **Mean** | 10 | 11 |
| **Variance** | 9 | 25 |
| **Sample Size** | 20 | 16 |

Does assembly line B have significantly more variability in the number of defects? Test at the $\alpha = 0.05$ significant level.

**Q.4** **(a)** Suppose we have a multinomial population with four categories: A, B, C, and D. The null hypothesis is that the proportion of items is the same in every category. The null hypothesis is **(07)**

$$H_0 : p_A = p_B = p_C = 0.25$$

A sample of size 300 yielded the following results.

$$A: 85 \quad B: 95 \quad C: 50 \quad D: 70$$

Use $\alpha = 0.05$ to determine whether $H_0$ should be rejected. What is the p-value?

**(b)** One of the questions on the *BusinessWeek* Subscriber Study was, "In the past 12 months, when traveling for business, what type of airline ticket did you purchase most often? " The data obtained are shown in the following contingency table. **(07)**

| | Type of Flight | |
|---|---|---|
| **Type of Ticket** | **Domestic Flights** | **International Flights** |
| **First class** | 29 | 22 |
| **Business/executive class** | 95 | 121 |
| **Full fare economy/coach class** | 518 | 135 |

M20-4

Use $\alpha = 0.05$ and test for the independence of type of flight and type of ticket. What is your conclusion?

**OR**

(a) Use the Kruskal-Wallis test to determine whether there is a significant difference in the following groups. Use $\alpha = 0.05$ **(07)**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **Group 1** | 19 | 21 | 29 | 22 | 37 | 42 | |
| **Group 2** | 30 | 38 | 35 | 24 | 29 | | |
| **Group 3** | 39 | 32 | 41 | 44 | 30 | 27 | 33 |

(b) A sample of 10 men was used in a study to test the effects of a relaxant on the time required to fall asleep. Data for 10 subjects showing the number of minutes required to fall asleep with and without the relaxant follow. Use $\alpha = 0.05$ level of significance and the Wilcoxon Signed Rank test to determine whether the relaxant reduces the median time required to fall asleep. What is your conclusion? **(07)**

| | **Relaxant** | | | **Relaxant** | |
|---|---|---|---|---|---|
| **Subject** | **No** | **Yes** | **Subject** | **No** | **Yes** |
| 1 | 15 | 10 | 6 | 7 | 5 |
| 2 | 12 | 10 | 7 | 8 | 10 |
| 3 | 22 | 12 | 8 | 10 | 7 |
| 4 | 8 | 11 | 9 | 14 | 11 |
| 5 | 10 | 9 | 10 | 9 | 6 |

**Q.5** Attempt any **SEVEN** out of **TWELVE:** **(14)**

**(1)** Define: Stratified Sampling

**(2)** Define: Point Estimator

**(3)** Define: Type – II error

**(4)** A statistician calculates a 95% confidence interval for μ when σ is known. The confidence interval is Rs. 10000 to Rs. 20000, what is the amount of the sample mean?

**(5)** The bottler of a certain soft drink claims their equipment to be accurate and that the variance of all filled bottles is 0.05 or less. The null hypothesis in a test to confirm the claim would be written as_____.

**(6)** A nonparametric method for determining the differences between two populations based on two matched samples where only preference data is required is the _____.

**(7)** The Spearman rank-correlation coefficient for 20 pairs of data when $\sum d_i^2 = 50$ is _____.

**(8)** Write the Condition for applying the Central Limit Theorem (CLT) which approximates the sampling distribution of the mean with a normal distribution.

**(9)** If we have normal populations with known population standard deviations σ1 and σ2, the confidence interval estimate for the difference between two population means $(\mu_1 - \mu_2)$ is_____.

**(10)** The number of degrees of freedom for the appropriate chi-square distribution in a test of independence is _____.

**(11)** Give the name of a non-parametric test that will be used to test the randomness.

**(12)** What is the range of chi square statistics?

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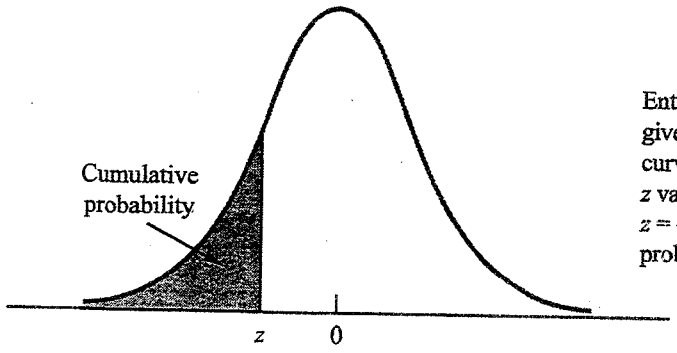
M70 – 5

# CUMULATIVE PROBABILITIES FOR THE STANDARD NORMAL DISTRIBUTION




Entries in this table give the area under the curve to the left of the $z$ value. For example, for $z = -.85$, the cumulative probability is .1977.

| z | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| −3.0 | .0013 | .0013 | .0013 | .0012 | .0012 | .0011 | .0011 | .0011 | .0010 | .0010 |
| −2.9 | .0019 | .0018 | .0018 | .0017 | .0016 | .0016 | .0015 | .0015 | .0014 | .0014 |
| −2.8 | .0026 | .0025 | .0024 | .0023 | .0023 | .0022 | .0021 | .0021 | .0020 | .0019 |
| −2.7 | .0035 | .0034 | .0033 | .0032 | .0031 | .0030 | .0029 | .0028 | .0027 | .0026 |
| −2.6 | .0047 | .0045 | .0044 | .0043 | .0041 | .0040 | .0039 | .0038 | .0037 | .0036 |
| −2.5 | .0062 | .0060 | .0059 | .0057 | .0055 | .0054 | .0052 | .0051 | .0049 | .0048 |
| −2.4 | .0082 | .0080 | .0078 | .0075 | .0073 | .0071 | .0069 | .0068 | .0066 | .0064 |
| −2.3 | .0107 | .0104 | .0102 | .0099 | .0096 | .0094 | .0091 | .0089 | .0087 | .0084 |
| −2.2 | .0139 | .0136 | .0132 | .0129 | .0125 | .0122 | .0119 | .0116 | .0113 | .0110 |
| −2.1 | .0179 | .0174 | .0170 | .0166 | .0162 | .0158 | .0154 | .0150 | .0146 | .0143 |
| −2.0 | .0228 | .0222 | .0217 | .0212 | .0207 | .0202 | .0197 | .0192 | .0188 | .0183 |
| −1.9 | .0287 | .0281 | .0274 | .0268 | .0262 | .0256 | .0250 | .0244 | .0239 | .0233 |
| −1.8 | .0359 | .0351 | .0344 | .0336 | .0329 | .0322 | .0314 | .0307 | .0301 | .0294 |
| −1.7 | .0446 | .0436 | .0427 | .0418 | .0409 | .0401 | .0392 | .0384 | .0375 | .0367 |
| −1.6 | .0548 | .0537 | .0526 | .0516 | .0505 | .0495 | .0485 | .0475 | .0465 | .0455 |
| −1.5 | .0668 | .0655 | .0643 | .0630 | .0618 | .0606 | .0594 | .0582 | .0571 | .0559 |
| −1.4 | .0808 | .0793 | .0778 | .0764 | .0749 | .0735 | .0721 | .0708 | .0694 | .0681 |
| −1.3 | .0968 | .0951 | .0934 | .0918 | .0901 | .0885 | .0869 | .0853 | .0838 | .0823 |
| −1.2 | .1151 | .1131 | .1112 | .1093 | .1075 | .1056 | .1038 | .1020 | .1003 | .0985 |
| −1.1 | .1357 | .1335 | .1314 | .1292 | .1271 | .1251 | .1230 | .1210 | .1190 | .1170 |
| −1.0 | .1587 | .1562 | .1539 | .1515 | .1492 | .1469 | .1446 | .1423 | .1401 | .1379 |
| −.9 | .1841 | .1814 | .1788 | .1762 | .1736 | .1711 | .1685 | .1660 | .1635 | .1611 |
| −.8 | .2119 | .2090 | .2061 | .2033 | .2005 | .1977 | .1949 | .1922 | .1894 | .1867 |
| −.7 | .2420 | .2389 | .2358 | .2327 | .2296 | .2266 | .2236 | .2206 | .2177 | .2148 |
| −.6 | .2743 | .2709 | .2676 | .2643 | .2611 | .2578 | .2546 | .2514 | .2483 | .2451 |
| −.5 | .3085 | .3050 | .3015 | .2981 | .2946 | .2912 | .2877 | .2843 | .2810 | .2776 |
| −.4 | .3446 | .3409 | .3372 | .3336 | .3300 | .3264 | .3228 | .3192 | .3156 | .3121 |
| −.3 | .3821 | .3783 | .3745 | .3707 | .3669 | .3632 | .3594 | .3557 | .3520 | .3483 |
| −.2 | .4207 | .4168 | .4129 | .4090 | .4052 | .4013 | .3974 | .3936 | .3897 | .3859 |
| −.1 | .4602 | .4562 | .4522 | .4483 | .4443 | .4404 | .4364 | .4325 | .4286 | .4247 |
| −.0 | .5000 | .4960 | .4920 | .4880 | .4840 | .4801 | .4761 | .4721 | .4681 | .4641 |

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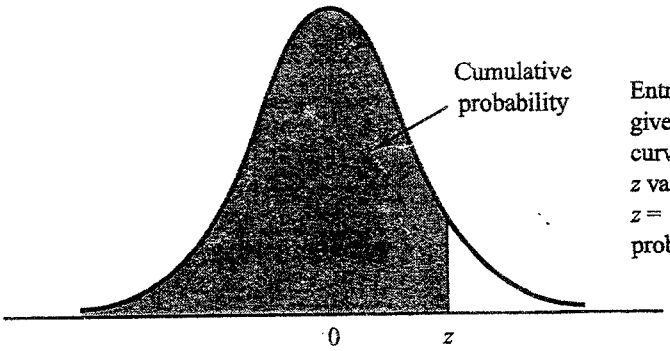
9

M170 – 6

## CUMULATIVE PROBABILITIES FOR THE STANDARD NORMAL DISTRIBUTION




Entries in the table give the area under the curve to the left of the $z$ value. For example, for $z = 1.25$, the cumulative probability is .8944.

| z | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| .0 | .5000 | .5040 | .5080 | .5120 | .5160 | .5199 | .5239 | .5279 | .5319 | .5359 |
| .1 | .5398 | .5438 | .5478 | .5517 | .5557 | .5596 | .5636 | .5675 | .5714 | .5753 |
| .2 | .5793 | .5832 | .5871 | .5910 | .5948 | .5987 | .6026 | .6064 | .6103 | .6141 |
| .3 | .6179 | .6217 | .6255 | .6293 | .6331 | .6368 | .6406 | .6443 | .6480 | .6517 |
| .4 | .6554 | .6591 | .6628 | .6664 | .6700 | .6736 | .6772 | .6808 | .6844 | .6879 |
| .5 | .6915 | .6950 | .6985 | .7019 | .7054 | .7088 | .7123 | .7157 | .7190 | .7224 |
| .6 | .7257 | .7291 | .7324 | .7357 | .7389 | .7422 | .7454 | .7486 | .7517 | .7549 |
| .7 | .7580 | .7611 | .7642 | .7673 | .7704 | .7734 | .7764 | .7794 | .7823 | .7852 |
| .8 | .7881 | .7910 | .7939 | .7967 | .7995 | .8023 | .8051 | .8078 | .8106 | .8133 |
| .9 | .8159 | .8186 | .8212 | .8238 | .8264 | .8289 | .8315 | .8340 | .8365 | .8389 |
| 1.0 | .8413 | .8438 | .8461 | .8485 | .8508 | .8531 | .8554 | .8577 | .8599 | .8621 |
| 1.1 | .8643 | .8665 | .8686 | .8708 | .8729 | .8749 | .8770 | .8790 | .8810 | .8830 |
| 1.2 | .8849 | .8869 | .8888 | .8907 | .8925 | .8944 | .8962 | .8980 | .8997 | .9015 |
| 1.3 | .9032 | .9049 | .9066 | .9082 | .9099 | .9115 | .9131 | .9147 | .9162 | .9177 |
| 1.4 | .9192 | .9207 | .9222 | .9236 | .9251 | .9265 | .9279 | .9292 | .9306 | .9319 |
| 1.5 | .9332 | .9345 | .9357 | .9370 | .9382 | .9394 | .9406 | .9418 | .9429 | .9441 |
| 1.6 | .9452 | .9463 | .9474 | .9484 | .9495 | .9505 | .9515 | .9525 | .9535 | .9545 |
| 1.7 | .9554 | .9564 | .9573 | .9582 | .9591 | .9599 | .9608 | .9616 | .9625 | .9633 |
| 1.8 | .9641 | .9649 | .9656 | .9664 | .9671 | .9678 | .9686 | .9693 | .9699 | .9706 |
| 1.9 | .9713 | .9719 | .9726 | .9732 | .9738 | .9744 | .9750 | .9756 | .9761 | .9767 |
| 2.0 | .9772 | .9778 | .9783 | .9788 | .9793 | .9798 | .9803 | .9808 | .9812 | .9817 |
| 2.1 | .9821 | .9826 | .9830 | .9834 | .9838 | .9842 | .9846 | .9850 | .9854 | .9857 |
| 2.2 | .9861 | .9864 | .9868 | .9871 | .9875 | .9878 | .9881 | .9884 | .9887 | .9890 |
| 2.3 | .9893 | .9896 | .9898 | .9901 | .9904 | .9906 | .9909 | .9911 | .9913 | .9916 |
| 2.4 | .9918 | .9920 | .9922 | .9925 | .9927 | .9929 | .9931 | .9932 | .9934 | .9936 |
| 2.5 | .9938 | .9940 | .9941 | .9943 | .9945 | .9946 | .9948 | .9949 | .9951 | .9952 |
| 2.6 | .9953 | .9955 | .9956 | .9957 | .9959 | .9960 | .9961 | .9962 | .9963 | .9964 |
| 2.7 | .9965 | .9966 | .9967 | .9968 | .9969 | .9970 | .9971 | .9972 | .9973 | .9974 |
| 2.8 | .9974 | .9975 | .9976 | .9977 | .9977 | .9978 | .9979 | .9979 | .9980 | .9981 |
| 2.9 | .9981 | .9982 | .9982 | .9983 | .9984 | .9984 | .9985 | .9985 | .9986 | .9986 |
| 3.0 | .9987 | .9987 | .9987 | .9988 | .9988 | .9989 | .9989 | .9989 | .9990 | .9990 |

M70-7

# Statistical Values Useful for the Paper

**Table 1: t table**

| | Signficance Level | | | | | |
|---|---|---|---|---|---|---|
| **DOF** | **0.2** | **0.1** | **0.05** | **0.025** | **0.01** | **0.005** |
| **5** | 0.92 | 1.476 | 2.015 | 2.571 | 3.365 | 4.032 |
| **6** | 0.906 | 1.44 | 1.943 | 2.447 | 3.143 | 3.707 |
| **7** | 0.896 | 1.415 | 1.895 | 2.365 | 2.998 | 3.499 |
| **8** | 0.889 | 1.397 | 1.86 | 2.306 | 2.896 | 3.355 |
| **60** | 0.848 | 1.296 | 1.671 | 2 | 2.39 | 2.66 |
| **61** | 0.848 | 1.296 | 1.67 | 2 | 2.389 | 2.659 |
| **62** | 0.847 | 1.295 | 1.67 | 1.999 | 2.388 | 2.657 |
| **63** | 0.847 | 1.295 | 1.669 | 1.998 | 2.387 | 2.656 |
| **64** | 0.847 | 1.295 | 1.669 | 1.998 | 2.386 | 2.655 |
| **85** | 0.846 | 1.292 | 1.663 | 1.988 | 2.371 | 2.635 |
| **86** | 0.846 | 1.291 | 1.663 | 1.988 | 2.37 | 2.634 |
| **87** | 0.846 | 1.291 | 1.663 | 1.988 | 2.37 | 2.634 |
| **88** | 0.846 | 1.291 | 1.662 | 1.987 | 2.369 | 2.633 |
| **89** | 0.846 | 1.291 | 1.662 | 1.987 | 2.369 | 2.632 |

**Table 2: Chi-Square Table**

| | Signficance Level | | | | |
|---|---|---|---|---|---|
| **DOF** | **0.1** | **0.05** | **0.025** | **0.01** | **0.005** |
| **1** | 2.706 | 3.841 | 5.042 | 6.635 | 7.879 |
| **2** | 4.605 | 5.991 | 7.378 | 9.21 | 10.597 |
| **3** | 6.251 | 7.815 | 9.348 | 11.345 | 12.838 |
| **4** | 7.779 | 9.488 | 11.143 | 13.277 | 14.86 |
| **5** | 9.236 | 11.07 | 12.832 | 15.086 | 16.75 |
| **6** | 10.645 | 12.592 | 14.449 | 16.812 | 18.548 |

**Table 3: F Table**

| | Signficance Level | | | |
|---|---|---|---|---|
| **DOF** | | Numerator DOF | | |
| **Denominator DOF** | **Area in Upper tail** | **10** | **15** | **20** |
| **18** | **0.1** | 1.98 | 1.89 | 1.84 |
| | **0.05** | 2.41 | 2.27 | 2.19 |
| | **0.025** | 2.87 | 2.67 | 2.56 |
| | **0.01** | 3.51 | 3.23 | 3.08 |
| **19** | **0.1** | 1.96 | 1.86 | 1.81 |
| | **0.05** | 2.38 | 2.23 | 2.16 |
| | **0.025** | 2.82 | 2.62 | 2.51 |
| | **0.01** | 3.43 | 3.15 | 3 |