2/106

1604N198

Candidate's Seat No :_________

M.Sc Semester-2 Examination

407

Physics

April-2024

Time : 2-30 Hours] **[Max. Marks : 70**

Quantum Mechanics II and Mathematical Physics-II

Q.1 (A) Prove that the wave function remains symmetric with variation of time. Obtain the symmetric and anti-symmetric wave functions from unsymmetrized wave function. [07]

(B) Discuss the Pauli Principle for a system of non-interacting indistinguishable particles. [07]

OR

Q.1 (A) Discuss Heisenberg picture and obtain the Heisenberg's equations of motion for an operator $\hat{A}_H$. Prove that these equations are identical with the corresponding canonical equations of Hamiltonian in classical mechanics. [07]

(B) Discuss the interaction picture to prove that the state vector in interaction picture can be determined by Schrodinger equation, while the operators obey the Heisenberg equation. [07]

Q.2 (A) Discuss the time dependent perturbation theory, and obtain expression for the matrix element of Hamiltonian H'_{mn}, when the perturbation is just switched on. [07]

(B) Prove that the coherent states are not orthogonal. Also prove that the coherent states are closest to the classical states. [07]

OR

Q.2 (A) Write expressions for the creation and annihilation operators. Hence prove that $[\hat{a}, \hat{a}^+] = 1$. Explain how the effect of $\hat{a}$ and $\hat{a}^+$ creates equally spaced eigen value ladder of unit steps. [07]

(B) Prove that the expectation value of number operator is positive real number for the coherent states. Prove that "the probability that there are ***n*** photons in the coherent state is given by Poisson distribution". [07]

Q.3 (A) Evaluate definite integral (I) using the residue theorem, where I = [07]

$$\int_0^{2\pi} \frac{\cos 3\theta}{5-4\cos\theta}\, d\theta$$

(B) Using Cauchy-Riemann's conditions, find out following functions are analytic or non-analytic [07]

(i) $\omega = f(z) = \cos h(z)$

(ii) $\omega = f(z) = e^z$

OR

Q.3 (A) Cauchy integral theorem-Discuss Cauchy integral theorem for simply and multiply connected region. [07]

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(B) (i) Find residue of $f(Z)$ at $Z = i$ [07]

(ii) Find residue of $f(Z) = \frac{Z}{(2Z+1)\,(5-Z)}$ at $Z = -\frac{1}{2}$ and $Z = 5$ using simple pole method

Q.4 (A) Using Neumann series method evaluate [07]

$$\emptyset(x) = \frac{5}{6}x + \frac{1}{2}\int_0^1 xt\,\emptyset(t)dt$$

(B) Show that [07]

$$\emptyset(x) = x - \int_0^x (t-x)\,\emptyset(t)\,dt = \sin hx$$

OR

Q.4 (A) Evaluate following function using Neumann series method [07]

$$\emptyset(x) = x + \lambda\int_0^1 xt\,\emptyset(t)dt$$

(B) Prove that [07]

$$\emptyset(x) = 1 + \int_0^x (t-x)\,\emptyset(t)\,dt = \cos x$$

Q.5 Answer in brief **Any Seven** questions from the following: (Each question is of **two** marks). [14]

(i) What is the basic difference among the Schrodinger picture, Heisenberg picture and Interaction picture?

(ii) Write the spin eigen functions of S_z for s =3/2.

(iii) Define indistinguishable particles.

(iv) Prove that $[\widehat{a^+}, \hat{n}] = -\,\widehat{a^+}$

(v) Prove that $|3\rangle = (3!)^{-1/2}\hat{a}^{+3}\,|0\rangle$

(vi) Prove that $[\hat{a}, \hat{n}] = \hat{a}$

(vii) Write uses of Cauchy-Riemann's conditions.

(viii) Find out real $u\,(x, y)$, and imaginary $v\,(x, y)$ of function f(Z)=1/Z

(ix) What is a holomorphic function?

(x) Write the homogeneous and non-homogeneous integral equations.

(xi) If $\emptyset(x) = 1 - 2\int_0^x t\,\emptyset(t)\,dt$ then find out $f(x), \lambda, a, b$ *and* $K\,(x,t)$

(xii) If $\emptyset(x) = x + \lambda\int_0^1 xt\,\emptyset(t)\,dt$ then find out $\emptyset_1(x)$

***** PAPER ENDS *****