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1601N1582

Candidate's Seat No :__________

IMSc CS Sem.-2 (ATKT) Examination
Linear Algebra
January-2024

Time : 3-00 Hours] **[Max. Marks : 70**

Instructions:

- **Write both the Sections in the separate answer book.**
- **Both Sections having equal weightage.**
- **Draw Diagrams wherever necessary.**
- **Make Assumptions wherever necessary.**

Section - I

Q-1 Attempt any of the **TWO** questions: **(14)**

(A) Evaluate the following partial derivatives up to second order of given functions at a point $(0,0)$:

$$f(x)=\begin{cases}\frac{xy(x^2-y^2)}{x^2+y^2}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$$

(B) Show that the $\mathbb{R}^+$ forms a real vector space under given operations: For $u,v\in\mathbb{R}^+$ and $\alpha\in\mathbb{R}$;

Vector addition: $u+v=u.v$

Scalar Multiplication: $\alpha.u=u^{\alpha}$

(C) Use the Gram-Schmidt process to orthonormalise the set of linearly independent vectors $\{(1,2,1),(-1,1,0),(5,-1,2)\}$.

Q-2 Attempt any of the **THREE** following questions: **(15)**

(A) Check whether the following subsets of vector space are subspace or not :

1) $S_0=\{(x,y)|x^2+y^2=1\}\subset\mathbb{R}^2$

2) $S_1=\{(x,y,z)|\,2x+3y-5z=1\}\subset\mathbb{R}^3$

(B) Find distance between $u=i-3j+4k$ and $v=3i+4j+7k$. Also, find the projection of u on v.

(C) If $u_1=\frac{yz}{x},u_2=\frac{zx}{y},u_3=\frac{xy}{z}$, then prove that $\frac{\partial(u_1,u_2,u_3)}{\partial(x,y,z)}=4$.

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(D) Verify Euler's theorem for the following function

$$f(x,y) = \frac{x^{1/4} + y^{1/4}}{x^{1/5} + y^{1/5}}$$

Q-3 Attempt any of the **THREE** questions: **(06)**

a) Define angle between two non-zero vectors.

b) Let L denotes any line not passing through origin in $\mathbb{R}^2$. Is the L subspace of $\mathbb{R}^2$? Justify.

c) Define homogeneous function with example.

d) Find first order partial derivative of the function $f(x,y) = x^3 + x^2y - xy^2 + y^3$.

Section – II

Q-4 Attempt any of the **TWO** questions: **(14)**

(A) Answer the following questions for the given matrix:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$$

1) Write down the order of matrix A, A^2, A^{10}.

2) Is the matrix Upper triangle of lower triangle? Justify.

3) Find the rank, trace and determinant of the matrix A.

(B) Diagonalize the following matrix:

$$B = \begin{bmatrix} 3 & 1 \\ 2 & 2 \end{bmatrix}$$

(C) Show that congruence relation of set of integer $\mathbb{Z}$ is an equivalence relation.

Q-5 Attempt any of the **THREE** following questions: **(15)**

(A) Show that $\mathbb{R} - \{1\}$ forms a group under the following operation where for all $a, b \in \mathbb{R} - \{1\}$; $a * b = a + b - ab$.

(B) Find eigen value and eigen vectors of the following matrix:

$$C = \begin{bmatrix} 3 & -4 \\ 2 & -6 \end{bmatrix}$$

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(C) Show that the following matrix D satisfies its own characteristic polynomial:

$$D = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

(D) Check whether the following matrix invertible or not. Find inverse of the matrix E if exists:

$$E = \begin{bmatrix} 2 & 3 & 1 \\ 4 & -2 & 0 \\ 6 & 5 & 7 \end{bmatrix}$$

Q-6 Attempt any of the **THREE** questions: **(06)**

a) Define group with example.

b) Give an example of matrix which is upper triangular as well as lower triangular.

c) (True/False): The sum of two symmetric matrices is again symmetric matrix.

d) Define characteristic polynomial for square matrix of order n.

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