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1006E439

Candidate's Seat No :________

M.Sc AIML & AIML (DS) Semester-2 Examination

Numerical Optimization

Time : 3-00 Hours] **June-2024** **[Max. Marks : 100**

Instructions: (1) All questions are compulsory.

(2) Each section carries 50 marks.

(3) Use of non-programmable scientific calculator is allowed.

SECTION-I

Q-1 Attempt all. 18 marks

(A) Let $f: C \to R$ be a twice differentiable function on an open convex set $C \subset R^n$. Then prove that f is convex if and only if its Hessian matrix is positive semi Definite for each $x \in C$. 08 marks

OR

(A) Let $S \subset R^n$ be a non empty, closed convex set and $y \notin S$ with minimum Distance from y. Then prove that x_0 is the minimizing point if and only If $(y - x_0)^T(x - x_0) \leq 0$, for all $x \in S$. 08 marks

(B) Define truncated Taylor's series expansion. Find truncated Taylor's series Of $f(x, y) = xe^{(x^2+y^2)}$. 10 marks

Q-2 Attempt any **two**. 16 marks

(A) Perform five iterations of the Golden section method to minimize $f(x) = x(x - 1.5)$ on [0, 1] with interval of uncertainty as 0.3. 08 marks

(B) Perform five iterations of Fibonacci search method to minimize $f(x) = max\left\{x^2, \frac{1-x}{2}\right\}$ over $[-1, 1]$. 08 marks

(C) Find critical points of (1) $f(x, y) = \sqrt{4y^2 - 9x^2 + 24y + 36x + 36}$ 08 marks

(2) $g(x, y) = x^2 + 2xy - 4y^2 + 4x - 6y + 4$.

Q-3 Attempt any **two.** 16 marks

(A) Prove that intersection of arbitrary collection of a convex set is a convex set. Also, prove that if C is a convex set then αC is also a convex set. 08 marks

(B) Prove that the set of all optimal solutions to convex programming problem Is convex. 08 marks

(C) Attempt any **four**. 08 marks

(1) Define convex set. Is the union of two convex set forms a convex set? Justify your answer briefly.

(2) Give an example of a convex and concave function.

(3) Give an example of affine set.

(4) Define stationary point and critical point.

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(5) Give an example of a linear and non-linear function on R^3.
(6) Define open set.

SECTION-II

Q-1 Attempt all. 18 marks

(A) Write the algorithm for Steepest decent method and further use it to minimize $f(x,y) = x - y + 2x^2 + 2xy + y^2$ starting from $(0,0)$. 08 marks

OR

(A) Use Conjugate gradient method to minimize 08 marks

$$f(x,y) = 5x^2 + 2y^2 - 2xy - 4x - 4y + 4, \quad x_0 = (0,0).$$

(B) Explain the development of Fletcher- Roove's method in detail with its algorithm. 10 marks

Q-2 Attempt any **two**. 16 marks

(A) Write the algorithm for interior penalty method and further use it to Minimize $f(x,y) = 0.8x^2 - 2^x$ subject to constraint $x \leq 4$. 08 marks

(B) Use exterior penalty method to minimize $f(x,y) = -xy$ subject to Constraint $g(x) = x + 2y - 4 \leq 0$. 08 marks

(C) Use Sequential quadratic programming algorithm to minimize $f(x,y) = x^3 + 2y^2 - 8xy$ subject to $g(x) = y^2 - 1 = 0,\ h(x) = xy - 4 \leq 0$. Perform two iterations. Start from (2, 3). 08 marks

Q-3 Attempt any **two.** 16 marks

(A) Apply Hookes- Jeeve's pattern search algorithm to minimize 08 marks

$$f(x,y) = x - y + 2x^2 + 2xy + y^2, \qquad x_0 = (0,0), \Delta x = \Delta y = 0.8.$$

(B) Apply Nelder- Maid algorithm to minimize $f(x,y) = (x-2)^2 + (y-3)^2$ With $v_1 = (0,0), v_2 = (1,0), v_3 = (0,1)$. 08 marks

(C) Attempt any **four**. 08 marks

(7) Find the gradient of the function $f(x,y,z) = x^3yz - 3xy^2z + xyz + 17$ At $(-2,4,1)$.
(8) Give an example of 3×3 positive semi definite matrix. Also, give an example Of 3×3 negative semi definite matrix.
(9) Write flow chart of Genetic algorithm.
(10) Calculate inverse of a matrix $\begin{bmatrix} 4 & 4 \\ 7 & -1 \end{bmatrix}$.
(11) What is the physical interpretation of Gradient?
(12) Write the algorithm for conjugate steepest decent method.

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