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0101E929

Candidate's Seat No :__________

MSc AIML/DS Sem.-1 Examination
Linear Algebra & Numerical Methods
January-2024

Time : 3-00 Hours] **[Max. Marks : 100**

Instructions:

- Write both the Sections in the separate answer book.
- Both Sections have equal weightage

SECTION - I

Q.1 Give short answers for any five: (10)

a) Assume that T is a linear transformation. Find the standard matrix of T. T : $R^2 \to R^2$ reflects a vector in the y-axis

b) Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, and b = $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$, $p = \begin{bmatrix} 7 \\ -1 \\ 2 \end{bmatrix}$.
Determine if b is in the the column space of A.
Determine if p is in Nul A.

c) Compute the quadratic form $x^T Ax$, when $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ and $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

d) What are singular values?

e) How is the matrix Σ constructed?

f) Which of the following belong to class NP-complete problems: Travelling Salesman, Vertex Cover Problem, Power Method, Matrix Operations

Q.2 a) i) What is class NP-complete? Give examples of NP-Complete problems. (10)
ii) What does NP-hard mean? Give examples of NP-Hard problems.

Q.2 b) Find a real root of the equation f(x) = $x^3 - x - 1 = 0$ between 1 and 2 using bisection method (10)

OR

Q.2 a) Find a real root of the equation $f(x) = x^3 - 2x - 5, x_0 = 1$ using Newton Raphson method. (10)

Q.2 b) Explain the advantages and disadvantages of False Position method (10)

Q. 3 Attempt any two (20)

a) How can we get a Truncated SVD?

b) Solve the systems using Gauss Seidel method

$$\begin{aligned} 12x_1 + 3x_2 - 5x_3 &= 1 \\ x_1 + 5x_2 + 3x_3 &= 28 \\ 3x_1 + 7x_2 + 13x_3 &= 76 \end{aligned}$$

Conduct 3 iterations, and choose $[x_1\ x_2\ x_3]$= [1 0 1] as your initial guess.

c) Solve the systems using Gauss Jordan method

$$\begin{aligned} x_1 + x_2 + x_3 &= 2 \\ x_1 + 3x_2 + 3x_3 &= 0 \\ x_1 + 3x_2 + 6x_3 &= 3 \end{aligned}$$

P.T.O

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SECTION - II

Q. 4 Answer any 2 of the following: (10)

a) Find the determinant of $B = \begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 5 \end{bmatrix}$

b) $Let\ A = \begin{bmatrix} 6 & 0 & 0 & 0 \\ 7 & 3 & 0 & 0 \\ 9 & 5 & 7.5 & 0 \\ 2 & 6 & 0 & -7.2 \end{bmatrix}$

i) What are the eigenvalues of A?

ii) What are the eigenvalues of A^T?

iii) Calculate determinant of A

c) Explain the following terms in brief:

i) Symmetric matrix

ii) Orthogonal matrices

iii) Rank of a matrix

iv) Orthonormal vectors

v) Ill-conditioned matrix

Q.5 a) Let $A = \begin{bmatrix} 1 & 0 \\ 4 & -3 \\ 5 & -1 \end{bmatrix}$, $B = \begin{bmatrix} -2 & 4 & -3 \\ 5 & 1 & 9 \\ 1 & 1 & -2 \end{bmatrix}$, $C = \begin{bmatrix} 0 & -1 & 6 \\ 3 & -2 & 5 \\ 1 & 0 & 4 \end{bmatrix}$, $D = \begin{bmatrix} 10 & -4 \\ 5 & 2 \\ 8 & -1 \end{bmatrix}$ (10)

i) (B + C)D

ii) $D^T A$

iii) $A \odot D$

iv) 4A

v) diag(B) and trace(B)

Q.5 b) Draw the graph of the below adjacency matrix. Use powers of adjacency matrices to determine the number of paths of length 3 between the given v_2 and v_4 . (10)

$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

OR

Q.5 a) Find the inverse of (10)

$A = \begin{bmatrix} 1 & 2 & 2 \\ 1 & 0 & 2 \\ 3 & 1 & -1 \end{bmatrix}$

Q.5 b) Solve the systems using Cramer's rule (10)

$$\begin{aligned} 2x_1 + 3x_2 - x_3 &= 4 \\ x_1 - 2x_2 + x_3 &= 6 \\ x_1 - 12x_2 + 5x_3 &= 10 \end{aligned}$$

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Q. 6 Attempt any two (20)

a) Using the power method, find the largest eigenvalue and the corresponding eigenvector of A = $\begin{bmatrix} 1.5 & 0 & 1 \\ -0.5 & 0.5 & -0.5 \\ -0.5 & 0 & 0 \end{bmatrix}$

Conduct 3 iterations, and choose $[x_1 \;\; x_2 \;\; x_3] = [1\ 1\ 1]$ as your initial guess.

b) The weather in Columbus is either good, indifferent, or bad on any given day. If the weather is good today, there is a 60% chance the weather will be good tomorrow, a 30% chance the weather will be indifferent, and a 10% chance the weather will be bad. If the weather is indifferent today, it will be good tomorrow with probability 0.40 and indifferent with probability 0.30. Finally, if the weather is bad today, it will be good tomorrow with probability 0.40 and indifferent with probability 0.50.

i) What is the stochastic matrix for this situation?

ii) Suppose there is a 50% chance of good weather today and a 50% chance of indifferent weather. What are the chances of bad weather tomorrow?

iii) Suppose the predicted weather for Monday is 40% indifferent weather and 60% bad weather. What are the chances for good weather on Wednesday?

iv) In the long run, how likely is it for the weather in Columbus to be good on a given day? Find the steady state vector.

c) For the matrix A = $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$

a) Find the characteristic polynomial, the eigenvalues and eigen vectors of the matrix

b) Diagonalize if possible