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0512E868

Candidate's Seat No :_____

IM.Sc AIML Sem.-3 (NEP) Examination

DSC-C-AIML-231-T

Statatistics for ML

Time : 2-00 Hours] **December-2024** **[Max. Marks : 50**

Instructions: All questions are compulsory. Use of non-programmable scientific calculator is allowed.

Q.1 **(a)** Consider the following probability density function **(05)**

$$f_X(x) = \begin{cases} kx, & 0 \le x < a, \\ k(2a - x), & a \le x \le 2a, \\ 0, & otherwise. \end{cases}$$

a. Find the value of k for which f is a probability density function.

b. Find mean and variance of X.

(b) A car rental agency has either 0,1,2,3,4, or 5 cars returned each day, with probabilities $\frac{1}{6}, \frac{1}{6}, \frac{1}{3}, \frac{1}{12}, \frac{1}{6}$ and $\frac{1}{12}$, respectively. Find the mean and the variance of the number of cars returned. **(05)**

OR

(a) The average weight of 19-year-old women in America is 148.6 pounds with a standard deviation of 23.9 pounds. What is the minimum percentage of 19-year-old women who weigh between 96.0 and 201.2 pounds? What is the maximum percentage of women who weigh more than 201.2 pounds? **(05)**

(b) A discrete random variable x has probability function $p_x(x), where$ **(05)**

$$p_x(x) = \begin{cases} k\left(\frac{1}{2}\right)^x, x = 1,2,3 \\ 0, \quad otherwise \end{cases}$$

a. Find the value of k

b. Find mean and variance of X.

c. Find the cumulative distribution function $F_x(x)$.

Q.2 **(a)** If the joint probability distribution of X and Y is given by **(05)**

$$f(x,y) = \frac{1}{30}(x + y) \ for \ x = 0,1,2,3 \ \& y = 0,1,2$$

a. Find $P(X \le 1, Y > 0)$

b. Find $P(X + Y \le 2)$

c. Find $P(X = 1, Y \le 2)$

(b) Join distribution of X and Y is given by: **(05)**

$$f(x,y) = 4xye^{-(x^2+y^2)}; x \ge 0, y \ge 0$$

For the above join distribution, find the conditional density of X is given $Y = y$

OR

(a) Given the joint probability distribution **(05)**

(P.T.O)

$$f(x,y,z) = \frac{xyz}{108} \quad for\ x = 1,2,3\ ;\ y = 1,2,3\ \&\ z = 1,2$$

a. Find the joint marginal probability distribution of X and Y.
b. Find the marginal distribution of X.

(b) Consider yet again the joint probability density function **(05)**

$$f(x_1,x_2) = x_1^2 + \frac{x_1x_2}{3} \quad 0 \le x_1 \le 1, 0 \le x_2 \le 2$$
$$= 0 \qquad\qquad Otherwise$$

Determined $E(x_2|x_1)$.

Q.3 **(a)** In January 2003, the American worker spent an average of 77 hours logged on to the Internet while at work. Assume the population mean is 77 hours, the times are normally distributed, and that the standard deviation is 20 hours. **(05)**

a. What is the probability that in January 2003 a randomly selected worker spent fewer than 50 hours logged on to the Internet?
b. What percentage of workers spent more than 100 hours in January 2003 logged on to the Internet?

(b) Airline passengers arrive randomly and independently at the passenger-screening facility at a major international airport. The mean arrival rate is 10 passengers per minute. **(05)**

a. Compute the probability of no arrivals in a one-minute period.
b. Compute the probability that three or fewer passengers arrive in a one-minute period.
c. Compute the probability of at least one arrival in a 15-second period.

OR

(a) A military commander wishes to destroy an enemy bridge. Each flight of plane he sends out has a probability of 0.8 of scoring a direct hit on the bridge. It takes four direct hits to completely destroy the bridge. If he can mount seven assaults before the bridge become tactically unimportant, what is the probability that the bridge will be destroyed? **(05)**

(b) The time between arrivals of vehicles at a particular intersection follows an exponential probability distribution with a mean of 12 seconds. **(05)**

a. What is the probability that the arrival time between vehicles is 12 seconds or less?
b. What is the probability that the arrival time between vehicles is 6 seconds or less?
c. What is the probability of 30 or more seconds between vehicle arrivals?

Q.4 **(a)** A population proportion is 0.40. A simple random sample of size 200 will be taken and the sample proportion will be used to estimate the population proportion. **(05)**

a. What is the probability that the sample proportion will be within ± 0.03 of the population proportion?
b. What is the probability that the sample proportion will be within ± 0.05 of the population proportion?

(b) How large a sample should be selected to provide a 95% confidence interval with a margin of error of 10? Assume that the population standard deviation is 40. **(05)**

OR

(a) The National Quality Research Center at the University of Michigan provides a quarterly **(05)**

measure of consumer opinions about products and services. A survey of 10 restaurants in the Fast Food/Pizza group showed a sample mean customer satisfaction index of 71. Past data indicate that the population standard deviation of the index has been relatively stable with $\sigma = 5$.

a. Using 95% confidence, what is the margin of error?
b. What is the margin of error if 99% confidence is desired?

(b) A simple random sample with $n = 54$ provided a sample mean of 22.5 and a sample standard deviation of 4.4. (Use t-table values- $t_{(0.05,53)} = 2.006$ and $t_{(0.025,53)} = 1.674$) **(05)**

a. Develop a 90% confidence interval for the population mean.
b. Develop a 95% confidence interval for the population mean.

Q.5 Attempt any **TEN** out of **TWELVE:** (Each carries 01 mark) **(10)**

(1) The weight of an object is an example of
a. a continuous random variable
b. a discrete random variable
c. either a continuous or a discrete random variable, depending on the weight of the object
d. either a continuous or a discrete random variable depending on the units of measurement

(2) Define: Conditional Expectation

(3) Write down the Chebyshev's Inequality.

(4) Suppose X is a random variable such that $E(X) = 3$ and $V(X) = 5$. In addition, $H(X) = 2X - 7$. what is the value of $E[H(X)]$ and $\mathrm{V}[H(X)]$?

(5) The probability density function for a uniform distribution ranging between 2 and 6 is_____.

(6) The expected value for a binomial distribution is _____.

(7) Define: Systematic Sampling

(8) Write down properties of point estimator.

(9) The height of the rectangle depicting a uniform distribution is the probability of each outcome and it same for all of the possible outcomes. True or False?

(10) For any continuous random variable, the probability that the random variable takes on exactly a specific value is _____.

(11) What is the difference between stratified and convenience sampling.

(12) A statistician calculates a 95% confidence interval for μ when σ is known. The confidence interval is Rs. 18000 to Rs. 22000, what is the amount of the sample mean?

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(P.T.O)

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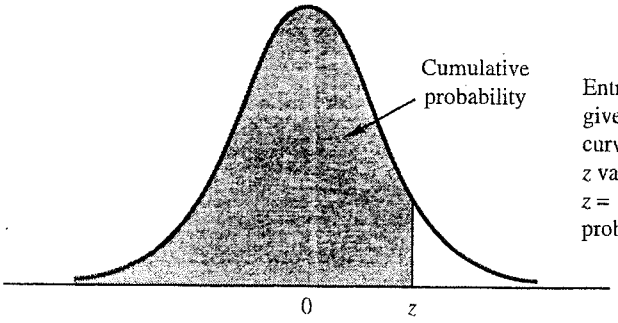
TABLE 1 CUMULATIVE PROBABILITIES FOR THE STANDARD NORMAL DISTRIBUTION (*Continued*)



Entries in the table give the area under the curve to the left of the z value. For example, for $z = 1.25$, the cumulative probability is .8944.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990

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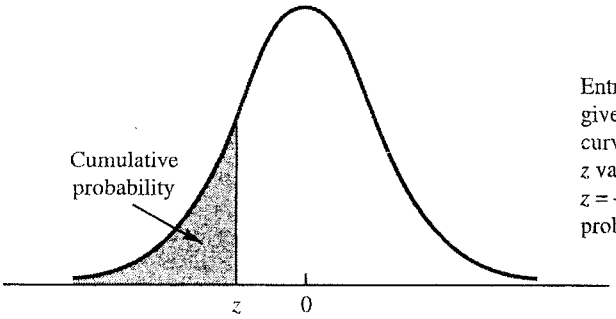
Appendix B: Tables

TABLE 1 CUMULATIVE PROBABILITIES FOR THE STANDARD NORMAL DISTRIBUTION



Entries in the table give the area under the curve to the left of the z value. For example, for $z = -.85$, the cumulative probability is .1977.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641