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0404N68

Candidate's Seat No :__________

M.Sc Semester-4 Examination

509

Mathematics (EA)

April-2024

Time : 2-30 Hours] **[Max. Marks : 70**

Instructions:

(1) All Questions are compulsory.

(II) Write the question number in your answer book as shown in the question paper.

(III) The figure to the right indicates marks of the question.

Q.1 **(A)** Prove that the highest power of a prime p which divide $n!$ is $\sum_{k=1}^{t}\left[\frac{n}{p^k}\right]$, where $p^t \leq n < p^{t+1}$. 7

(B) Define Mobius function. Also state and prove Mobius inversion formula. 7

OR

(A) Prove that every even perfect number is of the form $2^{k-1}(2^k - 1)$. 7

(B) Determine all solutions in the positive integers of the equation $54x + 21y = 906$. 7

Q.2 **(A)** State and prove Wilson's theorem. 7

(B) Solve: $2x \equiv 1(mod\ 5)$, $4x \equiv 1(mod\ 7)$, $5x \equiv 9(mod\ 11)$. 7

OR

(A) Show that for $k \geq 3$, the integer 2^k has no primitive root. 7

(B) Using indices for a primitive root of 11, solve the congruence $7x^3 \equiv 3(mod\ 11)$. 7

Q.3 **(A)** For odd prime p, prove that $(2/p) = \begin{cases} 1\ if\ p \equiv \pm 1(mod\ 8) \\ -1\ if\ p \equiv \pm 3(mod\ 8) \end{cases}$. 7

(B) If p is an odd prime and $(a,p) = 1$. Then prove that a is a quadratic residue of p if and only if $a^{(p-1)/2} \equiv 1(mod\ p)$. 7

OR

(A) For odd prime p with $(a,p) = 1$, prove that the congruence $x^2 \equiv a(mod\ p^n), n \geq 1$ has solution if and only if $(a/p) = 1$. 7

(B) Solve the congruence $x^2 \equiv 23\ (mod\ 7^3)$. 7

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N 68 · 2

Q.4 **(A)** Prove that value of any infinite continued fraction is an irrational number. 7

(B) Obtain all primitive Pythagorean triples x, y, z in which $x = 40$. 7

OR

(A) Define Pythagorean triangle. Prove that radius of the inscribed circle of a Pythagorean triangle is always an integer. 7

(B) By means of continued fractions determine the general solutions of Diophantine equation $19x + 15y = 1$. 7

Q.5 **Attempt any seven of the following.** 14

(1) The highest power of 3 which divides 31! is

(A) 10 (B) 12

(c) 13 (D) 14

(2) Values of $\sigma(n)$ and $\tau(n)$ for $n = 2700$ are

(A) 36 and 8680 (B) 36 and 868

(c) 12 and 8680 (D) 12 and 4340

(3) Which of the following equation is solvable in integers?

(A) $6x + 51y = 22$ (B) $11x + 22y = 121$

(c) $14x + 35y = 93$ (D) $21x + 28y = 58$

(4) Unit digit of 3^{100} is

(A) 3 (B) 9

(c) 1 (D) 7

(5) The remainder when 2(26!) divided by 29 is

(A) 0 (B) 1

(c) 26 (D) 28

(6) Order of 2 modulo 23 is

(A)11 (B) 12

(c)22 (D) none of these

(7) Which of the following congruence is not solvable?

(A) $x^2 \equiv 4(mod 13)$ (B) $x^2 \equiv 3(mod 17)$

(c) $x^2 \equiv 12(mod 13)$ (D) $x^2 \equiv 2(mod 17)$

N 68-3

(8) Index of 6 relative to primitive root 2 of 11 is

(A) 9 (B) 6

(c)5 (D) 4

(9) Which of the following integer is quadratic residue of 13?

(A) 6 (B) 8

(c) 7 (D) 9

(10) The finite simple continued fraction of $\frac{51}{19}$ is

(A) [3;2,1,2,6] (B) [2;1,2,6]

(c) [3;1,1,2,6] (D) none of these

(11) If (x, y, z) is a primitive Pythagorean tripie, then

(A) $\gcd(x, y) = 1$ (B) $\gcd(x, z) = 1$

(c) $\gcd(y, z) = 1$ (D) All of these

(12) The value of finite continued fraction $[0; 2, 1, 2]$ is

(A) $\frac{3}{8}$ (B) $\frac{6}{3}$

(c) $\frac{8}{3}$ (D) $\frac{4}{3}$

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0404N68 -4   Candidate's Seat No :________

M.Sc Semester-4 Examination

509

Mathematics (EB)

April-2024

Time : 2-30 Hours]   **[Max. Marks : 70**

Instructions: All questions are compulsory. Use of non-programmable scientific calculator is allowed.

Q.1 **(a)** Find Laplace transform of the function **(07)**

$$f(t) = \begin{cases} \frac{2t}{T}, & 0 \le t \le \frac{T}{2} \\ \frac{2}{T}(T-t), & \frac{T}{2} \le t \le T \end{cases} \qquad f(t+T) = f(t)$$

(b) Find the inverse Laplace transform of $\frac{1}{(s-2)(s^2+1)}$ by partial fraction method. **(07)**

OR

Q.1 **(a)** State the convolution theorem and find the inverse Laplace transform of $\frac{s^2}{(s^2+a^2)^2}$ **(07)**

(b) Using the Laplace transforms, find the solution of the initial value problem **(07)**

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = 0, \text{ where } y = 2, \frac{dy}{dx} = -4 \text{ at } x = 0$$

Q.2 **(a)** Find the Fourier series of $f(x) = x^2$ in the interval $(-\pi, \pi)$ and hence, deduce that **(07)**

$$\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \cdots$$

(b) Find Fourier transform of the function **(07)**

$$f(x) = \begin{cases} 1, & |x| < 1 \\ 0, & otherwise \end{cases}$$

OR

Q.2 **(a)** Find the eigenvalues and eigenfunctions of the Strum-Liouville problem: **(07)**

$$y'' + \mu y = 0, \qquad y(0) = 0, \qquad y(L) = 0$$

(b) Using Fourier Integral representation, show that **(07)**

$$\int_0^\infty \frac{\cos\omega x + \omega\sin\omega x}{1+\omega^2}\,d\omega = \begin{cases} 0 & x < 0 \\ \frac{\pi}{2} & x = 0 \\ \pi e^{-x} & x > 0 \end{cases}$$

Q.3 **(a)** Find Z-transform of $\sin\alpha k, k \ge 0$ **(07)**

(b) Find inverse Z-transform of $\frac{z}{(z-1)\,(z-2)}$ by residue method. **(07)**

OR

Q.3 **(a)** Solve the difference equation $6y_{k+2} - y_{k+1} - y_k = 0, y(0) = 0, y(1) = 1$ by Z-transform. **(07)**

(b) Find Z-transform of $\cosh\left(\frac{k\pi}{2} + \alpha\right)$ **(07)**

Q.4 **(a)** Evaluate $\int_0^a x\,(a^2 - x^2)\,J_0(sx)\,dx$ **(07)**

(b) I. Find the Hankel transform of $\frac{e^{-ax}}{x^2}, n = 1$ **(07)**

N 68.5

II. Find $H^{-1}[s^{-2}e^{-as}]$ when $n = 1$

OR

Q.4 **(a)** Show that if n = 0, the Hankel transform **(07)**

$$H\left\{\frac{sinax}{x}\right\} = \begin{cases} 0 & if\ s > a \\ \frac{1}{\sqrt{a^2-s^2}} & if\ 0 < s < a \end{cases}$$

(b) Prove Linearity property of Hankel transform and find the Hankel transform of **(07)**

$$f(x) = \begin{cases} 1 & 0 < x < a, \quad n = 0 \\ 0 & x > a, \quad n = 0 \end{cases}$$

Q.5 Attempt any **SEVEN** out of **TWELVE:** **(14)**

(1) Find Laplace transform of $t\cos at$

(2) Find Laplace transform of $sint\ u(t-4)$

(3) Find the inverse Laplace transform of $\frac{s+3}{s^2+2s+1}$

(4) Find the fundamental period of $\sin 2x, \cos 2\pi x$

(5) State Bessel's and Parseval's identity.

(6) Define: Fourier sine and cosine transform.

(7) Find Z-transform of discrete unit step $U(k) = \begin{cases} 0, & k < 0 \\ 1, & k \geq 0 \end{cases}$

(8) Find Z-transform of the sequence $\{a^k\}, k \geq 0$

(9) Write down the sequence $\{f(k)\}$ where $f(k) = \frac{1}{4^k}$, $-3 \leq k \leq 4$

(10) Define: Hankel transform

(11) Find the Hankel transform of $e^{-ax}, n = 1$

(12) Find $H^{-1}[e^{-as}]$ when $n = 0$

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