3009N605

Candidate's Seat No :________

B.Sc Sem.-6 (Rep) Examination
CC 310
Mathematics
September-2024

Time : 2-30 Hours] **[Max. Marks : 70**

Instruction: (1) **All Questions are compulsory.**
(II) **Figures to the right indicate full marks of the question.**

Q.1 (a) Prove that every $u - v$ walk contains a $u - v$ path, where u and v are any two vertices of a graph G. 7

(b) Define k-cube Q_k and prove that Q_k has 2^k vertices and $k2^{k-1}$ edges. 7

OR

Q.1 (a) State and prove first theorem of graph theory. Also prove that in any graph G there is an even number of odd vertices. 7

(b) Define Isomorphism of graphs. Give examples of isomorphic and non-isomorphic graphs. 7

Q.2 (a) Let G be a graph with n vertices and k connected components, then prove that G has atleast $n - k$ edges. 7

(b) Without drawing actual graph G, determine whether the graph is connected or not, where G is a graph whose adjacency matrix is 7

$$A(G) = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

OR

Q.2 (a) Prove that an edge e of a graph G is a bridge if and only if e is not a part of any cycle in G. 7

(b) In a Petersen graph G, find a trail of length 5, a path of length 9 and cycles of lengths 5, 6, 8 and 9. 7

Q.3 (a) Let G be a graph with n vertices $(n \geq 2)$. Then show that G has atleast two vertices, which are not cut vertices. 7

(b) Using Kruskal's algorithm find a minimal spanning tree of the following graph: 7

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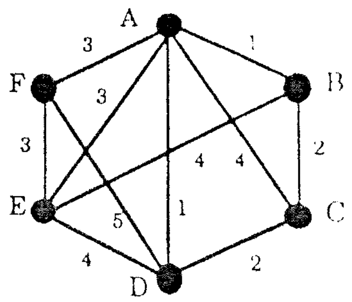
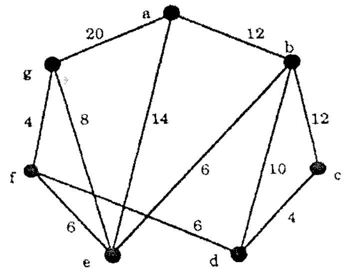


OR

Q.3 **(a)** Prove that if a graph G is connected, then it has a spanning tree. **7**

(b) Apply the Dijkstra's algorithm on the following connected weighted graph to find the length of shortest path from the vertex a to each of the other vertices of following graph : **7**



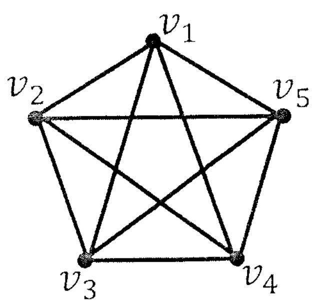
Q.4 **(a)** Let G be a graph in which the degree of every vertex is at least two. Then G contains a cycle. **7**

(b) Discuss the Koñigsberg bridges problem. **7**

OR

Q.4 **(a)** Prove that a simple graph G is Hamiltonian if and only if its closure is Hamiltonian. **7**

(b) Use the Fleury's algorithm to produce an Euler tour for the following graph: **7**



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Q.5 Attempt any seven short questions. **14**

(1) Define k-regualr graph and give an example of 3-regular graph.

(2) What is the radius of Petersen graph.

(3) What is the smallest positive integer n such that a complete graph K_n has atleast 600 edges?

(4) What is the determinant of adjacency matrix of cycle C_4.

(5) Let G be a connected graph with 17 edges. What is the maximum possible number of vertices in G?

(6) Draw a star graph $K_{1,5}$.

(7) Define cut vertex of a graph with an example.

(8) Define spanning tree.

(9) Define vertex connectivity of a graph.

(10) For which n, complete graph K_n is Euler?

(11) Define closure of a graph.

(12) Define Hamiltonian path and Hamiltonian cycle.