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Candidate's Seat No :________

M.Sc Semester-4 Examination

507

Mathematics

April-2024

Time : 2-30 Hours] **[Max. Marks : 70**

1. (A) Find the curvature and torsion of the helix $\bar{\gamma}(t) = (2\cos t,\ 2\sin t,\ 3t)$ $(0 \le t < \infty)$. 7

   (B) Let $\bar{\gamma}$ be a regular curve in $\mathbb{R}^3$ with never vanishing curvature. Prove that $\bar{\gamma}$ is planar if and only if the torsion of $\bar{\gamma}$ is identicall zero. 7

OR

1. (A) Define the tangent to a curve at a point. If $\bar{\gamma}$ be a smooth curve then prove that the tangent at the point $P = \bar{\gamma}(t_0)$ has the same direction as the vector $\bar{\gamma}'(t_0)$. 7

   (B) Find the curvature of the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$ at the vertices. 7

2. (A) Define the osculating paraboloid of a surface at a point. Find the equation of the osculating paraboloid to an ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ at the point $(0, 0, c)$, $c > 0$. 7

   (B) Make up the equation of the surface formed by straight lines parallel to a vector $\bar{v}$, and intersecting a curve $\bar{r} = \bar{r}(u)$. 7

OR

2. (A) Find the equation of the tangent plane to a sphere $\bar{r}(u, v) = (2\cos u \cos v,\ 2\cos u \sin v,\ 2\sin u)$ at the point $(2,\ 0,\ 0)$. 7

   (B) If a surface is given implicitly by the equation $\phi(x, y, z) = 0$ with $\phi_x^2 + \phi_y^2 + \phi_z^2 \neq 0$, find the equation of the tangent plane to the surface at a point (x_0, y_0, z_0). 7

3. (A) Show that the u and v curves are orthogonal on helicoid $\bar{r}(u, v) = (u\cos, u\sin v, v)$. 7

   (B) Define asymptotic on a surface. Find the asymptotic lines on the catenoid $x = \cosh u \cos v, y = \cosh u \sin v, z = u$. 7

OR

3. (A) Show that the surface area of a surface ϕ is given by $\iint_{\phi} \sqrt{EG - F^2}$. Show that the areas of the domain on the paraboloids $z = \frac{a^2}{2}(x^2 + y^2)$ and $z = axy$, projected onto the same domain of xy-plane, are equal. 7

   (B) Determine the principal curvature of a (hyperbolic) paraboloid $z = 2xy$ at the point $(0, 0, 0)$. 7

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4. (A) Define (i) geodesic (ii) a line of curvature on a surface. Show that if a geodesic is a line of curvature then it lies in a plane. **7**

(B) Prove that the sum of all three interior angles of a geodesic triangle on a sphere is greater than π. **7**

OR

4. (A) Define a geodesic on a surface. Determine a family of geodesics on the cylinder $S = \{(x, y, z)/x^2 + y^2 = 1\}$. **7**

(B) State (without proof) the second version (piecewise smooth curves version) of Gauss Bonnet theorem. Define the characteristic of a surface. **7**

5. **Attempt any SEVEN of the following:** **14**

(1) Which of the following is/are unit speed curve?

(A) $\bar{\gamma}(t) = (\cos 2t, \sin 2t)$
(B) $\bar{\gamma}(t) = (\cos t/2, \sin t/2)$
(C) $\bar{\gamma}(t) = (\frac{4}{5}\cos t, 1 - \sin t, -\frac{3}{5}\cos t)$
(D) $\bar{\gamma}(t) = (\cos^2 t, \sin^2 t,\ 0)$

(2) Let $\bar{\gamma}(t) = (e^t, t^2)$. Then the Cartesian form of the curve is

(A) $y = (lnx)^2$
(B) $y = (lnx^2)$
(C) $y = 2(lnx)^2$
(D) none of the above

(3) Consider the logarithmic spiral $\bar{\gamma}(t) = (e^t \cos t, e^t \sin t)$. What is the angle between $\bar{\gamma}(t)$ and $\bar{\gamma}'(t)$?

(A) $\cos^{-1}(\frac{1}{2})$
(B) $\cos^{-1}(\frac{1}{\sqrt{3}})$
(C) $\cos^{-1}(\frac{1}{\sqrt{2}})$
(D) $\cos^{-1}(\frac{1}{3})$

(4) What is the Dupin indicatrix of a surface at a hyperbolic point?

(A) A hyperbola
(B) Two conjugate hyperbolas
(C) Two parallel lines
(D) A hyperbolic paraboloid

(5) If all normals to a surface are concurrent, the surface must be ________

(A) catenoid
(B) spherical domain
(C) cylindrical
(D) helicoid

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(6) The normal curvature of a surface along ________ directions attains extreme values.

(A) asymptotic
(B) principal
(C) line of curvature
(D) none of the above

(7) Identify the surface $z^2 = x^2 + y^2$.

(A) Cone
(B) Parabolic cylinder
(C) Elliptic cylinder
(D) Elliptic paraboloid

(8) Which of the following are regular closed surfaces?

(A) A sphere
(B) A plane
(C) A torus
(D) A sphere with a deleted meridian.

(9) The Mean curvature and Gaussian curvature of a sphere of radius R (respectively) are ________

(A) $\frac{2}{R}, \frac{1}{R^2}$ (B) $\frac{1}{R}, \frac{1}{R^2}$ (C) $\frac{1}{R}, \frac{2}{R^2}$ (D) $\frac{1}{R}, \frac{1}{2R^2}$

(10) The shortest path on the surface $x^2 + y^2 + z^2 = 4$, between the points $(1, 1, \sqrt{2})$ and $(-1, -1, -\sqrt{2})$ has length

(A) 4π (B) π (C) 2π (D) 4

(11) Which of the following surfaces have constant negative Gaussian curvature?

(A) A surface of revolution
(B) Pseudo sphere
(C) A plane
(D) A torus

(12) What is the Euler's characteristic of sphere ?

(A) 1
(B) 2
(C) 3
(D) -2