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Candidate's Seat No :__________

M.Sc. Sem.-1 Examination

404

Mathematics

January-2024

Time : 2-30 Hours] **[Max. Marks : 70**

1. (A) Solve $y'' + 9y = 0$ in terms of power series in x. **7**

   (B) Solve $y'' + xy' + y = 0$ in terms of power series in x. **7**

OR

1. (A) Solve $y' + y = 1$ in terms of power series in x. **7**

   (B) Solve $(1 + x^2)y'' + xy' + y = 0$ in terms of power series in x. **7**

2. (A) Verify that the origin is a regular singular point and calculate two independent Frobenius series solutions for the equation $2x^2y'' + x(2x + 1)y' - y = 0$. **7**

   (B) Find the indicial equation and its roots for the following equations: **7**

   (1) $4x^2y'' + (2x^4 - 5x)y' + (3x^2 + 2)y = 0$

   (2) $x^3y'' - 4x^2y' + 3xy = 0$

OR

2. (A) Find the general solution of $(x^2 - 1)y'' + (5x + 4)y' + 4y = 0$ near the singular point $x = -1$. **7**

   (B) Determine the nature of the point $x = \infty$ for $x^2y'' + 4xy' + 2y = 0$. **7**

3. (A) Show that the Hermite functions are orthogonal on the interval $(-\infty, \infty)$. **7**

   (B) Obtain the recursion formula $(n + 1)\,P_{n+1}(x) = (2n + 1)\,xP_n(x) - nP_{n-1}(x)$ for the Legendre polynomials. Assume that $P_0(x) = 1$ and $P_1(x) = x$, calculate $P_2(x), P_3(x)$ and $P_4(x)$. **7**

OR

3. (A) Show that the Chebyshev polynomials are orthogonal on the interval $-1 \leq x \leq 1$ with respect to the weight function $(1 - x^2)^{-1/2}$. **7**

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(B) Prove that $\int_{-1}^{1} P_m(x)P_n(x)dx = 0$, if $m \neq n$. **7**

4. (A) Show that **7**

(1) $\frac{d}{dx}[x^p J_p(x)] = x^p J_{p-1}(x)$.

(2) $\frac{d}{dx}[x^{-p} J_p(x)] = -x^{-p} J_{p+1}(x)$.

(B) Show that **7**

(1) $\cos x = J_0(x) - 2J_2(x) + 2J_4(x) - \ldots$

(2) $\sin x = 2J_1(x) - 2J_3(x) + 2J_5(x) - \ldots$

OR

4. (A) Consider the initial value problem **7**

$$y' = 2y - 2x^2 - 3, \quad y(0) = 2.$$

Find successive approximations $y_0(x), y_1(x), y_2(x), y_3(x)$ using Picard's method.

(B) State Picard's theorem.(Do not prove.) **7**

Does $f(x, y) = xy^2$ satisfy a Lipschitz condition on any rectangle $a \leq x \leq b$ and $c \leq y \leq d$. Justify your answer.

5. **Attempt any seven of the following.** **14**

(1) The general solution of $y'' + y' - 2y = 0$ is

(A) $y = c_1e^{-x} + c_2e^{-2x}$ (C) $y = c_1e^{-2x} + c_2e^{3x}$

(B) $y = c_1 + c_2e^{-2x}$ (D) $y = c_1e^{x} + c_2e^{-2x}$

(2) The radius of convergence of $\sum_{n=0}^{\infty} (\frac{2}{3})^n (x)^{2n}$ is

(A) $\sqrt{3/2}$ (C) $\frac{3}{2}$

(B) $\sqrt{2/3}$ (D) ∞

(3) What are the ordinary points of the equation $x^2y'' + (sinx)y' = 0$?

(A) Each nonzero real x (C) only negative real x

(B) only positive real x (D) only $x = 0$.

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(4) For the equation $(x-1)^2y'' - 3(x-1)y' + 3y = 0$, the point $x = 1$ is

(A) an irregular singular point

(B) a regular singular point

(C) an ordinary point

(D) None of the above.

(5) The indicial equation of the differential equation $2x^2y'' + x(2x+1)y' - y = 0$ is

(A) $2m^2 + 2m + 1 = 0$

(B) $2m^2 + m - 1 = 0$

(C) $2m^2 - m - 1 = 0$

(D) $m^2 - 2m - 1 = 0$

(6) $\frac{d}{dx}F(1, b, b, x) =$ ______.

(A) $(1-x)^{-1}$

(B) $(1-x)^{-2}$

(C) $-(1-x)^{-1}$

(D) $-(1-x)^{-2}$

(7) Denote by $T_n(x)$ the n^{th} Chebyshev polynomial, $n = 0, 1, 2, \ldots$.
$T_3(x) =$ ______.

(A) $4x^3 - x$

(B) $x^3 - 3x^2$

(C) $4x^3 - 3x^2$

(D) $4x^3 - 2x^2 - 1$

(8) If $\sum_{n=0}^{\infty} a_nP_n(x)$ is a Legendre series of a function $f(x) = \begin{cases} 0 & \text{if } -1 \leq x < 0 \\ 2x+1 & \text{if } \ 0 < x \leq 1 \end{cases}$
then $a_2 =$ ______.

(A) 0

(B) $\frac{5}{4}$

(C) $\frac{5}{8}$

(D) $\frac{5}{2}$

(9) Which of the following are true for Legendre polynomial $P_n(x)$ of degree n?

(A) $P_n(1) = -1$

(B) $P_n(-1) = (-1)^n$

(C) $P_n(1) = 1$

(D) $P_n(-1) = 0$

(10) Which of the following are true for Bessel functions?

(A) $4J_2(x) - xJ_1(x) = xJ_3(x)$

(B) $J_{3/2}(x) = \sqrt{\frac{2}{\pi x}}\left(\frac{\sin x}{x} - \cos x\right)$

(C) $J_5(x)$ and $J_{-5}(x)$ are linearly dependent.

(D) $J_0'(x) = J_1(x)$

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(11) For $n = 1, 2, 3, \ldots, |J_n(x)|$_____.

(A) $\leq \frac{1}{\sqrt{2}}$

(B) $\geq \frac{1}{\sqrt{2}}$

(C) $< \frac{1}{\sqrt{2}}$

(D) $\leq \frac{1}{2}$

(12) The initial value problem corresponding to the integral equation $y(x) = 1 + \int_0^x y(t)dt$ is

(A) $y' - y = 0, y(0) = 1$

(B) $y' + y = 0, y(0) = 0$

(C) $y' - y = 0, y(0) = 0$

(D) $y' + y = 0, y(0) = 1$