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0101N1427

Candidate's Seat No :__________

M.Sc. Sem.-1 Examination

403

AMS

January-2024

Time : 2-30 Hours] **[Max. Marks : 70**

Instructions: All questions are compulsory. Use of non-programmable scientific calculator is allowed.

Q.1 **(a)** Verify that the origin is a singular point of $2x^2y'' + xy' - (x+1)y = 0$ and find two independent Frobenius series solutions of it. **(07)**

(b) Describe the nature of the critical point of the system and sketch the trajectory: **(07)**

$$\frac{dx}{dt} = -3x + 2y, \qquad \frac{dy}{dt} = -2x$$

OR

(a) For the initial value problem $\frac{dy}{dx} = y^2 + \cos^2 x, y(0) = 0$, determine the interval of existence of its solution given that R is the rectangle containing origin, **(07)**

$$R: \left\{(x,y):\ 0 \le x \le a,\quad |y| \le b,\quad a > \tfrac{1}{2},\quad b > 0\right\}$$

(b) Find all the eigenvalues and eigenfunctions of the Strum-Liouville problem: **(07)**
$Y'' + \lambda Y = 0$ with $y(0) + y'(0) = 0$ and $y(1) + y'(1) = 0$

Q.2 **(a)** Find the general integral of the following linear PDE: $(y + zx)p - (x + yz)q = x^2 - y^2$ **(07)**

(b) Find the characteristics of the equation $pq = z$ and determine the integral surface which passes through the straight line $x = 1, z = y$. **(07)**

OR

(a) Find the integral surface of the linear PDE: $xp - yq = z$, which contains circle: **(07)**
$x^2 + y^2 = 1, z = 1$.

(b) Find the complete integral of the equation $(p^2 + q^2)y = qz$. **(07)**

Q.3 **(a)** Reduce the PDE: $3\frac{\partial^2 z}{\partial x^2} + 10\frac{\partial^2 z}{\partial x \partial y} + 3\frac{\partial^2 z}{\partial y^2} = 0$ to canonical form. **(07)**

(b) Solve the Initial Boundary value problem for finite vibrating string: **(07)**
$u_{tt} = c^2 u_{xx}, 0 < x < l, t > 0$
with initial condition: $u(x,0) = f(x), 0 \le x \le l, u_t(x,0) = g(x), 0 \le x \le l$
and boundary conditions: $u(0,t) = 0 = u(l,t)$.

OR

(a) Solve: $(D^2 + DD' - 6D'^2)z = \sin(2x + y)$ **(07)**

(b) Solve Heat flow problem for semi-infinite rod: **(07)**
$u_t(x,t) = c^2 u_{xx}(x,t), 0 \le x < \infty, t > 0$
Boundary Condition: $u(0,t) = b_0, t > 0$,
Initial Condition: $u(x,0) = 0, 0 \le x < \infty$ with $u(x,t), u_x(x,t) \to 0$ as $x \to \infty$.

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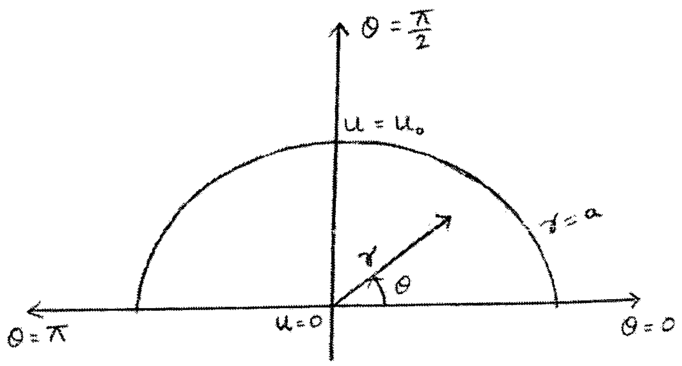
Q.4 **(a)** Find the steady state temperature distribution in a semi-circular plate of radius $'a'$, insulated on both the faces with a curved boundary kept at a constant temperature U_0 and its boundary diameter kept at zero temperature. **(07)**



Governing heat flow equation is $u_t = \nabla^2 u$

In steady state, temperature is independent of time,$u_t = 0$

To solve:

$$\text{P.D.E.: } \nabla^2 u(r,\theta) = u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0$$

$$\text{B.C.s: } u(a,\theta) = U_0,\ u(r,\theta) = 0,\ u(r,\pi) = 0$$

(b) **(i)** Prove that if the Dirichlet problem for a bounded region has a solution, then it is unique. **(07)**

(ii) Consider the parallel plate capacitor, where $V = 0$ at $z = 0$ and $V = 100v$ at $z = d$. Assuming the region between the plates charge-free. Calculate potential between the plates.

OR

(a) If u be a harmonic function in the interior of a rectangle $0 \le x \le a,\ 0 \le y \le b$ in the XY-plane satisfying Laplace equation **(07)**

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

with boundary conditions $u(0,y) = u(a,y) = u(x,b) = 0, u(x,0) = f(x)$

Obtain the solution to the above problem.

(b) Let $u(r,\theta)$ be the bounded solution of the following boundary value problem in polar coordinates: **(07)**

$$\nabla^2 u(r,\theta) = u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0,\ 0 < r < 2$$

$$u(2,\theta) = \cos^2\theta\,,0 \le \theta \le 2\pi$$

Then, find the value of $u\left(1,\frac{\pi}{2}\right) + u\left(1,\frac{\pi}{4}\right)$.

Q.5 Attempt any **SEVEN** out of **TWELVE:** **(14)**

(1) Give any (two) harmonic function which satisfies the Laplace Equation.

(2) State (only) ordinary point of the differential equation with suitable example.

(3) Show that $f(x,y) = x^2 + y^2$ on R: $|x| \le 1, |y| \le 1$ satisfy Lipschitz condition.

(4) Eliminate the arbitrary function from the following equation and hence, obtain the corresponding partial differential equation $x + y + z = f(x^2 + y^2 + z^2)$.

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(5) Find the complete integral of the following linear PDE: $x(1+y)p = y(1+x)q$.

(6) Show that the PDEs $xp - yq = x$ and $x^2p + q = xz$ are compatible or not.

(7) Determine that the p.d.e $yr + (x+y)s + xt = 0$ is hyperbolic, parabolic or elliptic.

(8) Write the complementary function for the non-homogeneous p.d.e. $DD'(D - 2D' - 3)z = 0$.

(9) Find the characteristic curves for one dimensional Wave equation.

(10) The force of attraction F both inside and outside the attracting matter, can be expressed in terms of a Gravitational Potential $'u'$ by the equation $F = \nabla u$. In empty space, $'u'$ satisfies _______ equation.

(11) Give any (two) harmonic function which satisfies the Laplace Equation.

(12) State (only) Neumann's problem for the rectangle.

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