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0101N1411

Candidate's Seat No :________

M.Sc. Sem.-1 Examination

403

Mathematics

January-2024

Time : 2-30 Hours]

[Max. Marks : 70

Instructions:

1. Use standard notations.
2. All questions are compulsory.
3. Mention question number clearly.

Q.1

(a) (i) Find modulus, conjugate, argument and principal argument for the complex number $Z=\dfrac{i}{2-\sqrt{3}i}$. 14

(ii) Check whether the function $f(z)=\dfrac{az+b}{cz+d}$ maps upper half plane into lower half plane or not in the following two cases.
$i)\ ad-bc>0 \quad ii)\ ad-bc<0$

OR

(b) (i) Find complex number Z if $\arg(Z+1)=\dfrac{\pi}{6}$ & $\arg(Z-1)=\dfrac{2\pi}{3}$. 14

(ii) If 'n' is any rational number then prove that
$(\cos\theta+i\sin\theta)^n=\cos(n\theta)+i\sin(n\theta)$.

Q.2

(a) (i) Determine the region of analyticity of following functions. 14

(1) $f(z)=\dfrac{(z-i)^2}{(z^2-z-1)(z+3i)}$ (2) $f(z)=\dfrac{(z+1)}{e^z-1}$.

(ii) Let $f(z)$ is analytic in some domain D which contains a line segment of the X-axis then $\overline{f(z)}=f(\bar{z})$ for each point z in the domain if and only if $f(x)$ is real for each point x on the segment.

OR

(b) (i) Derive the Cauchy-Riemann conditions in polar coordinates. 14

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(ii) Let $f(z)=\begin{cases}\dfrac{(\bar{z})^2}{z} & ;z\neq 0\\ 0 & ;z=0\end{cases}$ prove that $f(z)$ satisfy Cauchy-Riemann equations at $z=0$ but not differentiable at $z=0$.

Q.3

(a) (i) Find all the values of z such that $e^z=1+i$. 14

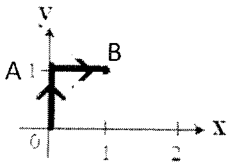
(ii) Find the value of the integral $I=\int_0^{2+2i} z^2 dz$ where contour C is along the parabolic path $y=2x^2$.

OR

(b) (i) (a) Find all the zeros of the given function $f(z)=\cos(2iz+13)$ 14

(b) Find the principal value of the function $(1-i)^{4i}$

(ii) Find $\int(y-x-i3x^2)\,dz$ over the contour C, where C is OAB in



Q.4

(a) (i) State and prove Cauchy Integral Formula. 14

(ii) Using Cauchy's integral formula, evaluate $\oint_{|z|=3} \dfrac{5z+1}{(z-1)(z-2)}dz$

OR

(b) (i) State and prove Liouville's Theorem. 14

(ii) Evaluate $\int_{C:|z|=1} \dfrac{\sin^6 z}{\left(z-\frac{\pi}{6}\right)^3}dz$.

Q.5 Attempt any **SEVEN** 14

1. Which of the following is a 4^{th} root of 'i'?

a) $2e^{i\frac{\pi}{8}}$ b) $e^{i\frac{\pi}{8}}$ c) $\sqrt{3}e^{i\frac{\pi}{8}}$ d) $\sqrt{3}e^{i\left(\frac{\pi}{8}+2k\pi\right)}$

2. Some zeros of $\sin(iz)$ lies on _________.

a) real axis b) imaginary axis c) both of above are correct d) none of the above

3. If the complex numbers $\sin x+i\cos 2x$ & $\cos x-i\sin 2x$ are conjugate to each other then what is the value of x?

a) $2k\pi, k\in\mathbb{Z}$ b) 0

c) $(2k+1)\frac{\pi}{2}, k \in \mathbb{Z}$ d) there does not exist any such 'x'

4. Let $f(z) = \bar{z}$. Then,

(a) $f(z)$ is analytic in the whole complex plane.

(b) $f(z)$ is analytic only at finitely many points in the complex plane.

(c) $f(z)$ is analytic nowhere in the complex plane.

(d) $f(z)$ is analytic only at the origin.

5. Let $f(z) = u(x, y) + iv(x, y)$ be analytic then, $f'(z) =$ ____.

(a) $u_x + iv_y$ (b) $u_y + iv_y$ (c) $v_x + iv_y$ (d) $v_y - iu_y$

6. Let $u(x, y)$ and $v(x, y)$ be such that u and v are harmonic conjugate of each-other. Then,

(a) u and v both are constant.

(b) u is constant and v is zero.

(c) u is zero and v is constant.

(d) u and v both are zero.

7. The principal value of i^{2i} is ____.

(a) $\exp(-\pi/2)$

(b) $\exp(\pi/2)$

(c) $\exp(-\pi)$

(d) $\exp(\pi)$

8. $\tanh^{-1}\left(\frac{1}{2}\right) =$ ____.

(a) $\log\sqrt{3/2}$

(b) $\log\sqrt{3}$

(c) $\frac{1}{2}\log\frac{3}{2}$

(d) $\frac{1}{2}\log\sqrt{3}$

9. $\cos^{-1} z =$ ____.

(a) $-i\log\left[z + i\left(1 - z^2\right)^{1/2}\right]$

(b) $-i\log\left[z - i\left(1 - z^2\right)^{1/2}\right]$

(c) $-i\log\left[z + i\left(1 - z^{1/2}\right)^2\right]$

(d) $i\log\left[z + i\left(1 - z^2\right)^{1/2}\right]$

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10. Find out the value of integration $I = \int_{C:|z|=10} \left(z^{10} + z^8 + z^2 + z + 1\right) dz$.

(a) 0
(b) 1
(c) 5
(d) $10^{10} + 10^8 + 10^2 + 10 + 1$

11. Consider four counters in positive direction as $C_1 : |z| = 1$, $C_2 : |z| = 5$, $C_3 : |z-3| = 1$ and $C_4 : |z - 3i| = 0.1$. Then which of the following is true for these contours?

(a) $C_1 = C_2 + C_3 + C_4$
(b) $C_2 = C_3 + C_4 + C_1$
(c) $C_3 = C_4 + C_1 + C_2$
(d) $C_4 = C_1 + C_2 + C_3$

12. Find out the value of $\int_{|z|=2} \frac{\exp(-5z)}{(z-1)^{10}} dz$.

(a) $(-5)^9$
(b) $(-5)^{10}$
(c) $(-5)^9 e^{-5}$
(d) $(-5)^{10} e^{-5}$

—x—