5/51 **0512E869** Candidate's Seat No :_____

IM.Sc DS Sem.-3 (NEP) Examination

DSC-C-DTSC-231-T

Statatistics for DS

Time : 2-00 Hours] **December-2024** **[Max. Marks : 50**

Instructions: All questions are compulsory. Use of non-programmable scientific calculator is allowed.

Q.1 **(a)** Consider the following probability density function **(05)**

$$f_X(x) = \begin{cases} kx, & 0 \le x < 2, \\ k(4-x), & 2 \le x \le 4, \\ 0, & otherwise. \end{cases}$$

a. Find the value of k for which f is a probability density function.
b. Find mean, variance of x.

(b) A used-car salesman find that he sells either 1, 2, 3, 4, 5 or 6 car per week with equal probability. **(05)**

a. Find the moment generating function of X.
b. Using moment generating function find $E(X)$ and $V(X)$.

OR

(a) The postal service requires, on the average, 2 days to deliver a letter across the town. The variance is estimated to be (0.4) $(days)^2$. If a business executive wants at least 99% of his letters delivered on time, how many should he mail them? **(05)**

(b) The continuous random variable T has the probability density function **(05)**
$f(t) = kt^2 \quad \text{for} -1 \le t \le 0.$

a. Find the appropriate value of k.
b. Find mean and variance of T.
c. Find the cumulative distribution function $F_x(x)$..

Q.2 **(a)** If the joint probability distribution of X and Y is given by **(05)**

$$f(x,y) = c(x^2 + y^2) \; for \; x = -1,0,1,3 \; \& \; y = -1,2,3$$

a. Find the value of c.
b. Find $P(X \le 1, Y > 2)$
c. Find $P(X + Y > 2)$

(b) Join distribution of X and Y is given by: **(05)**

$$f(x,y) = 4xye^{-(x^2+y^2)}; x \ge 0, y \ge 0$$

For the above join distribution, find the conditional density of X is given $Y = y$

OR

(a) Given the joint probability distribution **(05)**

$$f(x,y,z) = \frac{xyz}{108} \; for \; x = 1,2,3 \; ; \; y = 1,2,3 \; \& \; z = 1,2$$

a. Find the joint marginal probability distribution of X and Z.
b. Find the marginal distribution of Y.

(b) Consider yet again the joint probability density function **(05)**

(P.T.O)

$$f(x_1, x_2) = x_1^2 + \frac{x_1 x_2}{3} \quad 0 \le x_1 \le 1, 0 \le x_2 \le 2$$
$$= 0 \quad Otherwise$$

Determined $E(x_1|x_2)$.

Q.3 **(a)** The average stock price for companies making up the S&P 500 is Rs.30, and the standard deviation is Rs. 8.20. Assume the stock prices are normally distributed. **(05)**

a. What is the probability a company will have a stock price of at least Rs. 40?
b. What is the probability a company will have a stock price no higher than Rs. 20?

(b) During the period of time that a local university takes phone-in registrations, calls come in at the rate of one every two minutes. **(05)**

a. What is the probability of at least one calls in five minutes?
b. What is the probability of three calls in five minutes?
c. What is the probability of no calls in a five-minute period?

OR

(a) Five Cruise missiles have been built by an aerospace company. The probability of a successful firing is, on any test, 0.95. Assuming independent firing, what is probability that the first failure occurs on the fifth firing? **(05)**

(b) The time between arrivals of vehicles at a particular intersection follows an exponential probability distribution with a mean of 12 seconds. **(05)**

a. What is the probability that the arrival time between vehicles is 12 seconds or less?
b. What is the probability that the arrival time between vehicles is 6 seconds or less?
c. What is the probability of 30 or more seconds between vehicle arrivals?

Q.4 **(a)** A population has a mean of 200 and a standard deviation of 50. Suppose a simple random sample of size 100 is selected and is used to estimate μ. **(05)**

a. What is the probability that the sample mean will be within ± 5 of the population mean?
b. What is the probability that the sample mean will be within ± 10 of the population mean?

(b) How large a sample should be selected to provide a 95% confidence interval with a margin of error of 10? Assume that the population standard deviation is 40. **(05)**

OR

(a) In an effort to estimate the mean amount spent per customer for dinner at a major Atlanta restaurant, data were collected for a sample of 49 customers. Assume a population standard deviation of Rs.5. **(05)**

a. At 95% confidence, what is the margin of error?
b. If the sample mean is Rs. 24.80, what is the 95% confidence interval for the population mean?

(b) A simple random sample with $n = 54$ provided a sample mean of 22.5 and a sample standard deviation of 4.4. (Use t-table values- $t_{(0.05,53)} = 2.006$ and $t_{(0.025,53)} = 1.674$) **(05)**

a. Develop a 90% confidence interval for the population mean.
b. Develop a 95% confidence interval for the population mean.

Q.5 Attempt any **TEN** out of **TWELVE:** (Each carries 01 mark) **(10)**

(1) The number of customers that enter a store during one day is an example of
   a. a continuous random variable
   b. a discrete random variable
   c. either a continuous or a discrete random variable, depending on the number of the customers
   d. either a continuous or a discrete random variable, depending on the gender of the customers

(2) Define: Conditional Expectation

(3) Write down the Chebyshev's Inequality.

(4) Suppose X is a random variable such that $E(X) = 3$ and $V(X) = 5$. In addition, $H(X) = 2X - 7$. what is the value of $E[H(X)]$ and $V[H(X)]$?

(5) The probability density function for a uniform distribution ranging between 2 and 6 is_____.

(6) The standard deviation of a binomial distribution is _____.

(7) Define: Cluster Sampling

(8) Write down properties of point estimator.

(9) The height of the rectangle depicting a uniform distribution is the probability of each outcome and it same for all of the possible outcomes. True or False?

(10) For any continuous random variable, the probability that the random variable takes on exactly a specific value is _____.

(11) What is the difference between stratified and convenience sampling.

(12) A statistician calculates a 95% confidence interval for μ when σ is known. The confidence interval is Rs. 18000 to Rs. 22000, what is the amount of the sample mean?

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(P.T.O)

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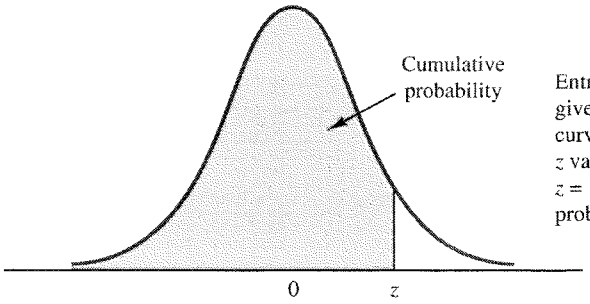
TABLE 1 CUMULATIVE PROBABILITIES FOR THE STANDARD NORMAL DISTRIBUTION (*Continued*)



Entries in the table give the area under the curve to the left of the z value. For example, for $z = 1.25$, the cumulative probability is .8944.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990

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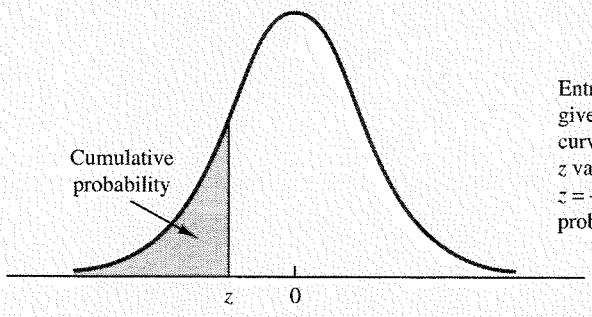
Appendix B: Tables

TABLE 1 CUMULATIVE PROBABILITIES FOR THE STANDARD NORMAL DISTRIBUTION



Entries in the table give the area under the curve to the left of the z value. For example, for $z = -.85$, the cumulative probability is .1977.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641