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0504N98

Candidate's Seat No :________

M.Sc Semester-4 Examination

510

Mathematics (EA)

April-2024

Time : 2-30 Hours] **[Max. Marks : 70**

Instructions:

1. All questions in Section I carry equal marks.
2. Attempt any 7 questions from Section II.

Section I

Q.1 A The John Equipment Company estimates its carrying cost at 10% and its ordering cost at \$90 per order. The estimated annual requirement is 60,,000 units at a price of \$4 per unit. Determine: 7

1. What is the most economical number of units to order?
2. How many orders should be placed in a year?
3. How often should an order be placed?

B A manufacturing concern requires 2,000 units of material per year. The ordering cost is Rs.20/order, while the carrying costs are Rs. 0.25 year/unit of the average inventory. The purchase price is Rs. 10 per unit. If a discount of 5 percent is available for orders of 100 units. If he purchases a single lot of 200 units, he has a discount of 10% per unit. What purchase quantity would you recommend? 7

OR

A If no shortages are allowed then determine the optimal and feasible order quantities for the following two products if there is a floor space restriction of 25,000 square feet. 7

Parameter	Product X	Product Y
Annual demand (Nos)	10000	15000
Ordering costs (\$/unit)	300	350
Unit cost (\$)	100	80
Carrying rate (% per year)	0.25	0.25
Space required (sq.ft.)	20	30

B A dealer supplies you the following information with regard to a product dealt by him: annual demand is 7,500 units, ordering cost is Rs. 20 per order, inventory carrying cost is 10% per unit per year of purchase cost of Rs. 200 per unit and cost of back-ordering will be Rs. 15 per unit per month. Find the 7

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maximum shortage permitted. How much additional cost will he have to incur on inventory if he does not permit back-ordering?

Q.2 A Derive steady-state probabilities (only) for ((M/M/1):(N/FCFS)) queue system. 7

B An arrival rate at a registration desk of a conference is considered to be Poisson with an average time of 5 minutes and exponential call lengths averaging 2 minutes. 7

1. Find the fraction of a day that the registration desk will be busy.
2. What is the probability that an arrival at the registration desk will have to wait?
3. What is the probability that an arrival will have to wait more than 10 minutes before the registration desk is free?
4. What is the probability that it will take him more than 10 minutes altogether to wait for the registration desk and complete his call?

OR

A Find waiting time distribution for system (M/M/1):(∞/FCFS). 7

B A group of engineers has two terminals to aid in their calculations. The average computing job requires 20 minutes of terminal time, and each engineer requires some computation, about once every 0.5 hour, i.e. the mean time between calls for service is 0.5 hours. Assume these are distributed according to an exponential distribution. If there are six engineers in the group, find the expected number of engineers waiting to use one of the terminals. 7

Q.3 A Player 1 can choose between two strategies, A or B, and Player 2 can also choose between two strategies, X or Y. The payoffs (in some arbitrary units) for player 1 are as follows: 7

		Player 2	
		X	Y
Player 1	A	3	1
	B	2	4

Assuming player 1 randomly chooses their strategy with an equal probability of 0.5, calculate the expected payoff for player 1 when player 2 chooses strategy X and Y.

B Without reducing the size , solve the following game graphically 7

		Player B	
		B_1	B_2
Player A	A_1	-2	5
	A_2	-5	3
	A_3	0	-2
	A_4	-3	0
	A_5	1	-4

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OR

A Solve the game by matrix method. 7

		Player B		
		B_1	B_2	B_3
	A_1	1	7	2
Player A	A_2	6	2	7
	A_3	5	1	6

B Solve the following game by Algebraic method 7

	$B_1(x_1)$	$B_2(x_2)$
$A_1(y_1)$	2	5
$A_2(y_2)$	3	1
$A_3(y_3)$	0	3

Q.4 A An investor wants to decide as to invest his money. He may invest in Bonds, Stocks or Mutual Funds and is shown in the table. 7

	Economy		
Alternatives	Growing	Stable	Declining
Bonda	50	45	5
Stocks	85	30	-10
Mutual Funds	62	35	-20

Using Laplace criterion in which the investor will decide to invest?

B Savri Company is evaluating an investment proposal that is uncertainly associated with three important aspects: the original cost, the useful life, and the annual net cash flows. The three probability distributions for these variables are shown below : 7

Original cost		Useful life		Annual net cash inflows	
Value	Probability	Period	Probability	Value	Probability
Rs.60,000	0.3	5 years	0.4	Rs.10,000	0.1
Rs.70,000	0.6	6 years	0.4	Rs.15,000	0.3
Rs.90,000	0.1	7 years	0.2	Rs.20,000	0.4
				Rs.25,000	0.2

The firm wants to perform five simulation runs of the project's life. The firm's cost of capital is 15% and the risk-free rate is 6%: for simplicity, it is assumed that these two values are known for certain and will remain constant over the life of the project.

To simulate the probability distribution of original cost, useful life, and annual net cash inflows, use the following sets of random numbers:

09, 84, 41, 92, 65; 24, 38, 73, 07, 04; and 07, 48, 57, 64, 72 respectively for each of the five simulation runs.

OR

A A construction company is planning to construct a complex there are three choices small complex, medium complex, or large complex. The pay-offs (in thousands of rupees) expected under various event action combinations, together with estimated probabilities of the likely demand are given here, 7

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Event	Probability	Small complex	Medium complex	Large complex
High demand (B_1)	0.4	1800	2200	4200
Low demand (B_2)	0.6	1000	600	-1200

Further, the company is looking to engage with a market research firm the record of the firm in terms of probability is

Event	A_1 - High demand report by firm	A_2 - Low demand report by firm
High demand (B_1)	0.9	0.1
Low demand(B_2)	0.2	0.8

The firm has asked Rs 3,00,000 as fee. How should the construction company proceed?

B Consider a store with one checkout counter. The customer arrives at this checkout counter at given intervals of time. The service time for each customer is given in the table. Prepare simulation table. Assume the arrival of the first customer at t=0. Estimate the average customer waiting and average server idle time. 7

Inter Arrival Time	3	2	6	4	4	5	8	7	-
Service Time	4	5	5	8	4	6	2	3	4

Section II

Q.5 Attempt any 7 14

1. The optimal order-level if the optimum lot-size is 109 units with a shortage cost per unit per year is Rs. 10 and holding cost is Rs. 2 per unit per year
   a. 90.83 b. 83.90 c. 83.83 d. 90.90
2. When a single quantity discount is applied, what happens to the total inventory costs as order quantity increases beyond the price break
   a. Costs increase
   b. Costs remain constant
   c. Costs decrease
   d. Costs fluctuate randomly
3. If a company having lot size 1448 units and production rate per year is 18,000 units then what is the manufacturing time
   a. 0.05 years
   b. 0.03 years
   c. 0.08 years
   d. 0.01 years
4. The state where the probability of the number of customers in the system is independent of time is known as
   a. Transient state b. Explosion state c. Jockeying d. Steady state
5. The probability density function is given as, $f(x) = 0.7e^{-0.7x}$, $x > 0$ then what is the value of mean and variance?
   a. $\frac{10}{49}, \frac{10}{7}$

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b. $\frac{10}{7}, \frac{100}{49}$
c. $\frac{100}{49}, \frac{10}{49}$
d. $\frac{100}{7}, \frac{10}{49}$

6. When the number of customers in the system is less than that of the servers then what will be the service rate?
   a. $\mu_n = n\mu$ b. $\mu_n = c\mu$ c. $\mu_n = \lambda\mu$ d. None

7. For the given payoff matrix, player A always uses

P-A \ P-B	I	II
I	-5	-2
II	10	5

   a. First strategy
   b. Mixed of both I and II
   c. Does not play game
   d. Second strategy

8. Find the probabilities of Machine A.

Machine A	Machine B	
	B_1	B_2
A_1	3	-2
A_2	2	2

   a. 2,3   b. 6,5   c. 0,1   d. 5,0

9. In a pay-off matrix, what do the axes of the graph represent in the graphical method?
   a. Player names   b. Strategies   c. Time Intervals   d. Pay-off values

10. To identify a problem, a manager ____.
    a. Compare one set of standards or goals to a second set of standards or goals
    b. Looks for unhappy customers
    c. Uses intuition to see that things don't look right
    d. Compares the current state of affairs with some standard or goal

11. For the following conditional payoff table, select the decision using EOL criterion.

State of nature	Course of action			Probability
	A1	A2	A3	
S1	35	40	-15	0.3
S2	40	25	30	0.4
S3	-20	60	40	0.3

12. The risk avoider has a __________ utility function.
    a. convex   b. concave   c. linear   d. either convex or concave

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SEM-IV (April 2024)

MATHEMATICS

MAT 510EA (Quantitative Techniques) OLD

Time: 2 ½ hours M.M. 70

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N18-7

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B An arrival rate at a registration desk of a conference is considered to be Poisson with an average time of 5 minutes and exponential call lengths averaging 2 minutes. 7

1. Find the fraction of a day that the registration desk will be busy.
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Q.3 A An engineering company is offered material handling equipment A. It is priced at Rs.40,000 including the cost of installation. The costs for operation and maintenance are estimated to be Rs 6,000 for each of the first five years, increasing every year by Rs.2,000 in the sixth and subsequent years. The company expects a return of 10 percent on all its investment. What is the optimal replacement period? 7

B A printing press has received five orders for printing. The table below shows the five jobs in the order of their arrival. Their processing time and due dates for five jobs A, B, C, D, and E are given in the table below. 7

Jobs	Processing Time (Days)	Due Dates (Days)
A	6	8
B	2	6
C	8	18
D	3	15
E	9	23

Obtain the optimal sequence according to the First come first serve (FCFS) rule and calculate

(1) Total completion time (2) Total processing time
(3) Average flow time (4) Average tardiness.

OR

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A A printing press has received five orders for printing . The table below shows the five jobs in the order of their arival. Their processing time and due dates for five jobs A, B, C, D and E are given in the table below. 7

Jobs	Processing Time (Days)	Due Dates (Days)
A	16	8
B	12	16
C	8	18
D	13	15
E	9	23

Obtain the optimal sequence according to First come first serve (FCFS) rule and calculate

(1) Total completion time (2) Total processing time

(3) Average flow time (4) Average tardiness.

B Using the graphical method, calculate the minimum time needed to process jobs 1 and 2 on five machines A, B, C, D and E. i.e. for each machine find the job that should be done first. Also calculate the total time needed to complete both jobs. 7

Job 1	Sequence	A	B	C	D	E
	Time(hrs)	3	4	2	6	2
Job 2	Sequence	B	C	A	D	E
	Time(hrs)	5	4	3	2	6

Q.4 A The automobile company manufactures around 100 scooters. The daily production varies from 96 to 104 depending upon the availability of raw materials and other working conditions: 14

production (per day) :	96	97	98	99	100	101	102	103	104
Probability :	0.04	0.09	0.12	0.14	0.11	0.10	0.20	0.12	0.08

The finished scooters are transported in a specially arranged lorry accommodating 150

scooters. Using following random numbers :

80, 81, 76, 75, 64, 43, 18, 26, 10, 12, 65, 68, 69, 61, 57.

Simulate the process to find out:

(1) What will be the average number of scooters waiting in the factory?

(2) What will be the average number of empty space on the lorry?

OR

A The investment corporation wants to study the investment projects based on three factors, namely, market demand in units price per unit minus cost per unit, and investment required. These factors are believed to be independent of each other. In analyzing a new consumer product, the corporation estimates the following probability distributions: 14

Annual demand		Price minus cost per unit		Investment required	
Units	probability	Rs.	Probability	Rs.	probability

22,000	0.20	2.00	0.20	13,50,000	0.25
29,000	0.30	4.00	0.30	15,00,000	0.50
36,000	0.10	6.00	0.50	20,00,000	0.25
43,000	0.10				
50,000	0.30				

Using the simulation process, repeat the trial 5 times, compute the return on investment for each trial taking these factors into account. What is the most likely return?

Section II

Q.5 Attempt any 7 14

1. The optimal order-level if the optimum lot-size is 109 units with a shortage cost per unit per year is Rs. 10 and holding cost is Rs. 2 per unit per year
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   b. $\frac{10}{7}, \frac{100}{49}$
   c. $\frac{100}{49}, \frac{10}{49}$
   d. $\frac{100}{7}, \frac{10}{49}$
6. When the number of customers in the system is less than that of the servers then what will be the service rate?
   a. $\mu_n = n\mu$ b. $\mu_n = c\mu$ c. $\mu_n = \lambda\mu$ d. None
7. An optimal solution to n jobs through three machines can be obtained if of the following conditions hold good.
   a. min $t_{1j} \geq$ max t_{2j} b. min $t_{3j} \geq$ max t_{2j}
   c. A and B both d. either or both
8. In 2 Jobs by m machine sequencing, a line 45° represents:
   a. Job 1 is idle
   b. Job 2 is idle
   c. No Job is idle
   d. both Jobs are idle
9. The time between the start of the first job and completion of the last job in sequencing problems is called ___
   a. Elapsed time. b. Total time.
   c. Assignment time. d. Idle time.

For Q. 10-12

A dairy keeps a stock of a popular brand of chees. Daily demand based on past experience is given below:

Daily Demand	0	10	24	25	21	8
Probability	0.01	0.12	0.08	0.16	0.28	0.35

Consider the following sequence of random numbers:
28, 30, 16, 25, 51, 56, 60, 22, 34, qnd 68.

10. What is the random number interval at the demand 21?
    a. 13 – 20
    b. 21 – 36
    c. 01 – 12
    d. 37 – 64
11. The average daily demand for the cheese block:
    a. 22 b. 23 c. 24 d. 25
12. The total stock of cheese block left
    a. 33 b. 32 c. 30 d. 31

(P.T.O)

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Candidate's Seat No :________

M.Sc Semester-4 Examination

510

Mathematics (EB)

April-2024

Time : 2-30 Hours] **[Max. Marks : 70**

1. (A) Define self-adjoint operator on H. Prove that operators 0 and I are self-adjoint.

   (B) Show that $T \in \beta(H)$ is unitary if and only if T is an isometric isomorphism of H onto itself. 7

OR

1. (A) Define normal operator on H. Give an example of a normal operator that is not self-adjoint.

   (B) If $T \in \beta(H)$ is self-adjoint, prove that the inner product (Tx, x) is a real number for each $x \in H$. 7

2. (A) State (without proof) finite dimensional spectral theorem. 7

   (B) If P and Q are projections on H, under what conditions $P+Q$ becomes a projection on H? 7

OR

2. (A) Define the spectrum $\sigma(T)$ of T. Find the spectrum of $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x, y) = (2x, 2y)$. 7

   (B) If P and Q are projections on closed linear subspaces M and N of H respectively, prove that $P \leq Q \Leftrightarrow M \subseteq N$ 7

3. (A) For any $A \in BL(X)$, define the spectrum $\sigma(A)$, eigen spectrum $\sigma_e(A)$ and the approximate eigen spectrum $\sigma_a(A)$ of A. 7

   (B) Give the right shift operator R on l^2. Find eigen spectrum of R. 7

OR

3. (A) State (without proof) Gelfand-Mazur theorem. 7

   (B) Find the eigen spectrum of $A(x) = (x_1, \frac{x_2}{2}, \frac{x_3}{3}, \ldots)$, where $x \in l^2$. 7

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4. (A) Define compact linear map. Give an example of a continuous linear map that is not comapct. 7

(B) Show that the identity map on l^2 is not compact. 7

OR

4. (A) Prove that the space $CL(X, Y)$ of all compact linear maps from X to Y is a linear space. 7

(B) Prove that any linear map T from $\mathbb{R}^3 \to \mathbb{R}^3$ is compact. 7

5. **Attempt any SEVEN of the following:** 14

(1) Which of the following spaces is a Hilbert space in its usual norm?

(A) l^2	(C) l^3
(B) l^1	(D) l_∞

(2) What is the dimension linear space C over $\mathbb{R}$?

(A) 1	(C) 4
(B) 2	(D) infinite

(3) If $P \in \beta(H)$ is a projection then which of the following statements are true?

(A) P is positive	(C) P is idempotent
(B) P is linear	(D) none of the above

(4) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be given by $T(x, y) = (2y, 3x)$. Then T is ________

(A) linear	(C) unbounded
(B) continuous	(D) none of these

(5) Let G denote the set of all invertible matrices in algebra $M_{2\times2}(\mathbb{R})$. Then ________

(A) G is closed	(C) G is empty set
(B) G is open	(D) none of the above

(6) Let $T \in \beta(H)$ be isometry. Then ________

(A) T is one-one	(C) T is invertible
(B) T is onto	(D) none of these

(7) Let H be finite dimensional and $T \in \beta(H)$ such that $T^3 = 0$. Then

(A) $\sigma(T) = \{0\}$
(B) $\sigma(T)$ may contain a non-zero scalar.
(C) $\sigma(T)$ is empty.
(D) none of these

(8) Can we have an operator T on H such that the spectrum of T, $\sigma(T) = (0, 1)$?

(A) Yes
(B) Yes, if H is infinite dimensional
(C) No
(D) Yes, if H is finite dimensional

(9) Let $f \in X = (C[0, 1], ||\cdot||_\infty)$ be defined by $f(x) = x$, for each $x \in [0, 1]$. What is the spectrum $\sigma(f)$ of f?

(A) $\sigma(f) = \{0\}$
(B) $\sigma(f) = \{0, 1\}$
(C) $\sigma(f) = [0, 1]$
(D) $\sigma(f) = (0, 1)$

(10) What is the spectral radius $r_\sigma(T)$ of the linear continuous map $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by $T(x, y, z) = (x,\ y,\ 0)$?

(A) 0 (B) 1 (C) 2 (D) 3

(11) Let X be a sequence space which is a Banach space. Let (α_n) be a sequence of scalars converging to 0. For each $x \in X$, let $A(x_1,\ x_2,\ x_3, ...) = (0,\ k_1 x_1,\ k_2 x_2,\ ...)$. Then __________

(A) $\sigma(A) = \{0\}$
(B) $\sigma_e(A) = \{0\}$
(C) $\sigma(A) = \{\alpha_n / n = 1, 2, ...\}$
(D) none of these

(12) Let $T : C[0, 1] \to \mathbb{R}$ be defined by $T(f) = f(\frac{1}{2})$, for each $f \in C[0, 1]$. Then

(A) T is a functional
(B) T is compact
(C) T is not continuous
(D) none of these