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Candidate's Seat No :__________

M.Sc. Sem.-1 Examination

404

AMS

January-2024

Time : 2-30 Hours] **[Max. Marks : 70**

Instructions: All questions are compulsory. Use of non-programmable scientific calculator is allowed.

Q.1 **(a)** Find all the possible Jordan canonical forms of the matrix A whose order is 6×6 defined over R (set of all real numbers) with characteristic polynomial $(x-3)^2(x-2)^4$ and minimal polynomial $(x-3)(x-2)^2$. **(07)**

(b) **(i)** Let $A = \begin{bmatrix} 7 & 2 \\ -4 & 1 \end{bmatrix}$. Find a formula for A^k, given that $A = PDP^{-1}$, where $P = \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}$ and $D = \begin{bmatrix} 5 & 0 \\ 0 & 3 \end{bmatrix}$ **(07)**

(ii) Show that the following matrix is not diagonalizable.

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$$

OR

(a) In a certain town, 30 percent of the married women get divorced each year and 20 percent of the single women get married each year. There are 8000 married women and 2000 single women, and the total population remains constant. Investigate the long-range prospects if these percentages of marriages and divorces continue indefinitely into the future. **(07)**

(b) Let $A = \begin{pmatrix} 0.95 & 0.03 \\ 0.05 & 0.97 \end{pmatrix}$. Analyze the long-term behavior of the dynamical system defined by $x_{k+1} = Ax_k$ $(k = 0,1,2,\dots)$, with $x_0 = \begin{pmatrix} 0.6 \\ 0.4 \end{pmatrix}$ **(07)**

Q.2 **(a)** Find the QR decomposition of matrix $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 0 \\ 0 & -1 & 1 \end{bmatrix}$ **(07)**

(b) Let R^3 have the inner product $\langle (x_1, x_2, x_3), (y_1, y_2, y_3) \rangle = x_1y_1 + 2x_2y_2 + 3x_3y_3$. Use the Gram-Schmidt process to transform the basis vectors $u_1 = (1,1,1), u_2 = (1,1,0), u_3 = (1,0,0)$ into an orthonormal basis. **(07)**

OR

(a) Given the quadratic equation $5x^2 - 4xy + 8y^2 - 36 = 0$ find a change of coordinates so that the resulting equation represents a conic in standard position. Sketch the rough figure of the required conic. **(07)**

(b) Determine the Algebraic Multiplicity and Geometric Multiplicity of each eigenvalue of the following matrix: **(07)**

$$A = \begin{pmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{pmatrix}$$

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Q.3 **(a)** Factorize $A = \begin{pmatrix} 2 & -2 & 4 \\ 2 & 3 & 2 \\ -1 & 1 & -1 \end{pmatrix}$ by Crout's method. **(07)**

(b) Find the Singular Value Decomposition of the matrix $A = \begin{bmatrix} 3 & 1 & 1 \\ -1 & 3 & 1 \end{bmatrix}$. **(07)**

OR

(a) Determine $\|.\|_F$, $\|.\|_\infty$, and $\|.\|_1$ for each of the following matrices: **(07)**

(i) $\begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix}$ **(ii)** $\begin{pmatrix} 5 & 0 & 5 \\ 4 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}$

(b) Find the Singular Value Decomposition (SVD) of the matrix: **(07)**

$$A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix}$$

Also, find the closest (with respect to Frobenius norm) matrix of rank 1.

Q.4 **(a)** Prove that $\mathbb{N}$ (a set of nature numbers) with respect to divisibility relation forms a lattice. **(07)**

(b) If L is distributive complemented lattice, then show that $(a \oplus b)' = a' * b'$. **(07)**

OR

(a) Let $(L, \leq)$ be a lattice. For any $a, b, c \in L$ prove that, $a \leq c \Leftrightarrow a \oplus (b * c) \leq (a \oplus b) * c$. **(07)**

(b) Minimize the following Boolean expression using K-map **(07)**

$$f(A, B, C, D) = \sum m(0,1,5,7,8,10,14,15)$$

Q.5 Attempt any **SEVEN** out of **TWELVE:** **(14)**

(1) State (only) Diagonalization of the matrices with suitable example.

(2) Express the following quadratic forms in matrix notation:

$$2x^2 + 3y^2 - 5z^2 - 2xy + 6xz - 10yz$$

(3) Check whether the following matrices are similar or not.

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \text{ and } B = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

(4) State the necessary condition for Gauss-Seidel iterative method with suitable example.

(5) Identify the curve of the quadratic equation: $x^2 + 4xy + y^2 + 4x + 6y + 4 = 0$.

(6) Determine $\|.\|_F$ *and* $\|.\|_\infty$ for each of the following matrices: $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

(7) State (only) Orthonormal set with suitable example.

(8) State (only) the difference between the algorithm of Crout's and Doolittle method.

(9) Determine norm $\|A\|_1$, where $A = \begin{bmatrix} 1 & -2 \\ 3 & 1 \end{bmatrix}$.

(10) Consider $X = \{1,2,3\}$ and power set of X as $P(X)$ with respect to set inclusion relation. Does a set inclusion relation on $P(X)$ forms a partially ordered relation? Justify your answer.

(11) Draw Hasse diagram of (S_{45}, D).

(12) Prove that: $(A + B)(A + C) = A + BC$

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