0101N1412 Candidate's Seat No :__________

M.Sc. Sem.-1 Examination

403

Statistics

January-2024

Time : 2-30 Hours] **[Max. Marks : 70**

Note: Attempt all questions.

Q - 1 (a) What is Sufficient statistic? State and prove Neyman`s factorization criterion for discrete case. **[07]**

Q - 1 (b) Let x be a random sample from N(μ,σ^2), where μ &σ^2 are unknown. Obtain minimal sufficient statistic for μ &σ^2 . **[07]**

OR

Q - 1 (a) Define completeness of a statistic. Let x be a random sample frombinomial b(n , p) , 0<p<1 , check whether it is complete or not? **[07]**

Q - 1 (b) If T=(T1, T2, T3, ...Tk) be a complete statistic and U =(U1, U2, U3, ... Uk) be another statistic with relationship U = f(T) which is one-one function between U and T, then show that U is also Complete. **[07]**

Q - 2 (a) State and prove Rao-Blackwell theorem. **[07]**

Q - 2 (b) Let x_1 and x_2 be independent & identically distributes Poisson random variables. **[07]**

a.) Show that W = $\begin{cases} 1, if\ x_1 = 0 \\ 0,\ otherwise \end{cases}$ is an unbiased estimator of $g(\theta) = e^{-\theta}$.

b.) Compute $E[W / x_1 + x_2 = y]$

c.) For estimator W in a.), find a uniformly better unbiased estimator of $e^{-\theta}$

OR

Q - 2 (a) State and prove Lehmann-Scheffe theorem for UMVUE. **[07]**

Q - 2 (b) Let x1 and x2 be two unbiased estimators of **θ** such that $V(x_1) = V(x_2)$ & $T=k_1x_1 + k_2x_2(k_1 + k_2 = 1)$. Find out real constants k1 & k2 such that T becomes MVUE of **θ**. **[07]**

Q - 3 (a) State Cramer Rao Inequality and the regularity conditions. Also define MVBUE. **[07]**

Q - 3 (b) Let x follows N(μ,σ^2), σ^2 *is known.* Obtain MVBUE for μ. Also obtain its variance. **[07]**

OR

Q - 3 (a) Define MLE. State its properties and prove any one of them. **[07]**

Q - 3 (b) Let (x1,x2,...,xn) be a random sample from log normal distribution LN(μ,σ^2) , x>0 , $-\infty < \mu < \infty$.Obtain MLE of mean and variance of the distribution. **[07]**

$f(x,\mu,\sigma) = \frac{1}{\sqrt{2\pi}\sigma x} \exp\{\frac{-1}{2\sigma^2}(\log x - \mu)^2\}$

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N1412-2

Q - 4 (a) Explain the method of minimum chi-square and modified minimum chi-square. **[07]**

Q - 4 (b) Obtain the asymptotic confidence interval for unknown parameter θ from a distribution having p.d.f $f(x,\theta)$, having MLE $\hat{\theta}$. **[07]**

OR

Q - 4 (a) Explain Bayes Risk and Bayes Decision Rule. Using the extensive form of Bayesian estimation, obtain the Baye`s d-rule for squared error loss function. **[07]**

Q - 4 (b) Let {x1, x2, ... ,xn} be a random sample of size n from poissondistribution with mean θ , $\theta > 0$. Prior distribution of θ is $\pi(\theta) = \frac{e^{-\alpha\theta}\theta^{p-1}\alpha^{p}}{\Gamma p}$, $\theta > 0, \alpha > 0, p > 0$ then obtain Baye`s estimate of θ. **[07]**

Q - 5 Answer any seven: **[14]**

(i) If f(x1,x2,...xn; θ)=g(t ;θ) h(x1,x2,....,xn), then t is__________

a)Sufficient statistic b)Consistent statistic
c)Efficient statistic d)All of these

(ii) If T = $(\sum_{i=1}^{n} xi , \sum_{i=1}^{n} xi^2)$ is minimal sufficient for (μ, σ)
then we can claim that $(\bar{x}, s^2)$ is also Minimal sufficient statistic? Why?

(iii) Let $X1$,$X2$,$X3$ be iid from $U(0,\theta)$.
a)min ($x1$,$x2$,$x3$) is sufficient statistic for θ
b)max ($x1$,$x2$,$x3$) is sufficient statistic for θ
c)($X(1)$,$X(3)$,) is jointly sufficient statistic for θ
d)($x1$+$x2$ + $x3$) is sufficient statistic for θ

(iv) Let x~$Fx(\frac{x}{\theta})$. Here any statistic made from (n-1) ratios is

a)Ancillary Statistic b) unbiased estimator
c)complete sufficient statistic d)None of these

(v) Absolute error loss function will be minimum under
a)Posterior mean
b)Posterior standard deviation
c)Posterior median
d)None of these

(vi) Let $X1$,$X2$,....Xn be iid from $U(-\theta,\theta)$, then the maximum MLE for θ is

a)Max($-x_{(1)}$,$x_{(n)}$) b)Min ($-x_{(1)}$,$x_{(n)}$)
c)$x_{(n)}$ d)$x_{(1)}$

(vii) Describe concept of Bayes principal.

(viii) Explain exponential family of distributions.

(ix) For large values of n, method of minimum chi-square reduces to method of maximum likelihood estimation. True or False?

(x) State the Invariance property of MLE.

(xi) Does MLE always exist? If not, give a counter example.

(xii) Define method of Scoring to obtain MLE.

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