Seat No. : _______________

# AL-101

**April-2023**

## B.Sc., Sem.-IV

## 204 : Statistics

## Distribution Theory – II

**Time : 2:30 Hours]** **[Max. Marks : 70**

**Instruction :** Each sub-question carries **7** marks and question **5** carries **1** mark each.

1. Write the following : **14**

   (i) Define negative binomial distribution. Derive the CGF of negative binomial distribution. Hence determine find first four cumulants.

   (ii) State and prove the memoryless property for geometric distribution.

   **OR**

   (i) Define Geometric distribution. Obtain MGF of geometric distribution. Hence determine mean and variance.

   (ii) Prove that Poisson distribution is a limiting case of Negative Binomial distribution.

2. Write the following : **14**

   (i) Define Cauchy distribution. Derive the distribution function and hence determine the median of Cauchy distribution.

   (ii) Define Log-normal distribution. Determine quartiles of log-normal distribution.

   **OR**

   (i) Define Laplace distribution. Derive the moment generating function of Laplace distribution.

   (ii) Define two parameter Weibull distribution. Derive the distribution function and hence determine median of Weibull distribution.

3. Write the following : **14**

(i) Define a normal distribution. Derive the mgf of normal distribution. Hence determine its mean and variance.

(ii) Let (X, Y) ~ BVND $\left(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho\right)$. Show that X and Y are independent if and only if $\rho = 0$.

**OR**

(i) Let (X, Y) ~ BVND $\left(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho\right)$. Obtain the conditional distribution of X given Y = y.

(ii) Let X ~ N $(\mu, \sigma^2)$. Show that all the odd order moments of X are zero. Also find the expression for even order central moments.

4. Write the following : **14**

(i) State and prove the Lindberg-Levy form of central limit theorem.

(ii) With the usual notation prove that

$$\mu_r' = (-i)^r \left[\frac{d^r \phi x(t)}{dt^r}\right]_{(t=0)}$$

**OR**

(i) State and prove Bernoulli's Law of Large numbers.

(ii) Define Characteristics function. Derive the characteristics function of geometric distribution.

5. Attempt any **seven** out of **twelve**. **14**

(i) Give the probability mass function of Hypergeometric distribution.

(ii) Write two different forms of negative binomial distribution.

(iii) Let the random variable X denote the number of attempts to get first success in independent Bernoulli's trials. What is the probability mass function of X ?

(iv) Two independent random variables $X_1$ and $X_2$ have same geometric distribution, what is the distribution of $X_1$ given $X_1 + X_2$ ?

(v) State the pdf of standard Log-normal distribution.

(vi) Let X and Y be independent Log-normal. What is the distribution of XY ?

(vii) State the mean and variance of Laplace distribution.

(viii) Let (X, Y) ~ BVND (0, 0, 1, 1, $\rho$). Write the conditional pdf of X given Y = y.

(ix) State the formula for mean deviation about mean of normal distribution.

(x) Define standard normal distribution.

(xi) Define Convergence in Probability.

(xii) State the characteristics function of Binomial distribution.

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