Seat No. : _______________

AP-102

April-2023

M.Sc., Sem.-IV

510 : Mathematics

(EA : Quantitative Techniques)

Time : 2:30 Hours] **[Max. Marks : 70**

Instructions : (1) All questions in Section **I** carry equal marks.

(2) Attempt any **7** questions from Section **II**.

Section I

1. (A) The probability distribution of monthly sales of a certain item is as follows : **7**

Weekly sales	0	1	2	3	4	5	6	7	8
Probability	0.01	0.05	0.09	0.15	0.30	0.22	0.13	0.03	0.02

The cost of carrying inventory is ₹ 10.00 per unit per week. The current policy is to maintain a stock of 4 items at the beginning of each week. Assuming that the cost of shortage is proportional to both time and quantity short, obtain the range of shortage cost of short unit.

(B) A company producing three items has a limited storage space of an average of 1000 items of all types. Determine the optimal production quantities for each item separately, when the following information is given : **7**

Cost	Product		
	1	2	3
Holding cost (₹)	0.5	0.2	0.4
Setup cost (₹)	60	30	70
Demand (units)	300	500	100

OR

(A) Find the optimal order quantity for a product for which the price-breaks are as under : **7**

Quantity	Unit cost (₹)
$0 \le Q_1 < 200$	10.00
$200 \le Q_2 < 400$	9.00
$Q_3 \ge 500$	8.00

The monthly demand for the product is 100 units. The cost of storage is 4% of the unit cost and the cost of ordering is ₹ 250 per order.

(B) The cost of parameters and other factors for a production inventory system of paint colour are given below. Find (a) optimal lot size, (b) number of shortage and (c) manufacturing time and time between set-ups. **7**

Demand per year = 6000 units

Units cost = ₹ 40

Set-up cost ₹ 500

Production rate per year = 36000 units

Holding cost per year = ₹ 8

Shortage cost per unit per year ₹ 20

2. (A) Derive steady state probabilities (only) for ((M/M/l):(∞/FCFS)) queue system. **7**

(B) A group of engineers has two terminals to aid in their calculations. The average computing job requires 20 minutes of terminal time, and each engineer requires some computation, about once every 0.5 hour, i.e. the mean time between calls for service is 0.5 hours. Assume these are distributed according to an exponential distribution. If there are six engineers in the group, find the probability of free terminal. **7**

OR

(A) In a factory cafeteria the customers have to pass through three counters. The customers buy coupons at the first counter, select and collect the snacks at the second counter and collect tea at the third. The server each counter takes on an average 15 minutes although the distribution of service time is approximately Poisson at an average rate of 6 per hour. Calculate **7**

(a) The average time a customer spends waiting in the cafeteria

(b) The average time of getting the service

(c) The expected waiting time of a customer in the system

(B) If a customer arrives at a book stall in accordance with Poisson process with mean rate of 3 per minute. Find probability that the inter-arrival time between two consecutive arrivals is **7**

(a) More than 1 minute

(b) Between 1 and 2 minutes

(c) 4 min. or less

3. (A) Assume that the present value of one rupee to be spent in a year's time is ₹ 0.9 and C = ₹ 4,000, capital cost of equipment. The running costs are given in the table below : **7**

Year	1	2	3	4	5	6	7
Running Costs (in ₹)	400	500	700	900	1100	1400	1800

When should the machine be replaced ?

(B) Six jobs have to be processed at three machines A, B and C in the order BCA. The time taken by each job on each machine is indicated below. Each machine can process only one job at a time. **7**

	Job	J_1	J_2	J_3	J_4	J_5	J_6
Processing time in minutes on machines	A	12	8	7	11	10	5
	B	3	4	2	5	1.5	4
	C	7	10	9	6	10	4

Determine the sequence of the jobs so as to minimize the processing time.

OR

(A) Let the value of money be assumed to be 10% per year and suppose that machine A is replaced after every 3 years, whereas machine B is replaced after every 6 years. The yearly costs (in ₹) of both the machines are given below : **7**

Year	1	2	3	4	5	6
Machine A	1000	200	400	1000	200	400
Machine B	1700	100	200	300	400	500

Determine which machine should be purchased.

(B) Six jobs 1, 2, ..., 6 are to be processed on a single machine. The processing times and due dates of the jobs are represented in below table. **7**

Jobs	Processing Time (Days)	Due Dates (Days)
1	5	15
2	8	10
3	6	13
4	3	20
5	10	22
6	14	40

Then obtain optimal sequence according to SPT (Shortest processing time) rule and calculate

(1) Average flow time

(2) Average tardiness

(3) Number of jobs actually late.

4. A job has to be processed over two machines M_1 and M_2 in that order. The distribution of inter-arrival time of the job at the first machine is as follows : **14**

Time (min.)	1	2	3	4
Probability	0.3	0.15	0.25	0.3

The processing times at the two machines are as follows :

Machine M_1		Machine M_2	
Time (min.)	Probability	Time (min.)	Probability
1	0.2	4	0.3
2	0.1	5	0.4
3	0.2	6	0.2
4	0.4	7	0.1
5	0.1	-	-

On the basis of 10 simulation runs, find out the average queue length before machine M_1 and the average queue length before machine M_2.

Use the following random numbers:

For inter-arrival time 8, 7, 8, 9, 4, 4, 0, 6, 8, 7

For processing time at the machine M_1 0, 6, 9, 3, 7, 0, 0, 5, 6, 4

For processing time at the machine M_2 3, 0, 0, 0, 0, 2, 4, 5, 1, 2

OR

A wholesaler stocks an item for which demand is uncertain. He wishes to assess two re-ordering policies, i.e., order 12 units at a re-order level of 12 units or order 10 units at a re-order level of 10 units, to see which is most economical over a 10-day period. The following information is given : **14**

Demand per day (units)	:	4	5	6	7	8
Probability	:	0.10	0.14	0.26	0.20	0.30

Carrying cost is ₹ 10 per unit per day, ordering cost is ₹ 70 per order, loss of goodwill cost per each unit out of stock is ₹ 30, lead-time is 3 days and opening stock is of 17 units.

The probability distribution is to be based on the following random numbers :

41, 92, 05, 44, 66, 07, 00, 00, 14, 62 (for policy 1)

20, 07, 95, 15, 79, 95, 64, 26, 06, 48 (for policy 2)

Section II

5. Attempt any 7 : **14**

1. The cycle time is the time
   (A) to place orders for material.
   (B) time of receiving material.
   (C) the time between receipt of material and using the material.
   (D) the time between placing the order and receiving the material.

2. What is optimal order-level if optimum lot-size is 109 units with shortage cost per unit per year is ₹ 10 and holding cost is ₹ 2 per unit per year ?
   (A) ₹ 77.37 (B) ₹ 83.76
   (C) ₹ 110.85 (D) ₹ 90.83

3. The time period between placing an order and its receipt in stock is known as
   (A) Lead time (B) Carrying time
   (C) Shortage time (D) Over time

4. The congestion exists in a queue if
   (A) $\lambda > \mu$. (B) $\lambda < \mu$.
   (C) $\lambda \leq \mu$. (D) $\lambda \geq \mu$.

5. The number of faults per month that arise in the gearboxes of buses is known to follow Poisson distribution with means 2.5 faults per month. What is the probability that in a given month no faults are found ?

6. Define Balking in queueing system.

7. The following failures have been observed for a certain type of transistors in a digital computer :

End of week	1	2	3	4	5	6	7	8
Probability of failure to date	0.05	0.13	0.25	0.43	0.68	0.88	0.96	1.00

   Total number of transistors at the beginning of assembly is 1000 units. The cost of replacing the individual failed transistors is ₹ 1.25. The decision is made to replace all these transistors simultaneously at fixed intervals and to replace the individual transistors as they fail in service. The cost of group replacement is 30 paisa per transistors. Find the expected life of each transistor.
   (A) 9.69 weeks (B) 7.45 weeks
   (C) 3.72 weeks (D) 4.62 weeks

8. An automobile unit requires 200 junior engineers, 300 assistant engineers and 50 executives. Trainees are recruited at the age of 21 years, if still in service; retire at the age of 60. Given the following life table, determine how many engineers should be recruited each year.

Age	21	22	23	24	25	26	27	28
No. of persons in service	1000	600	400	384	307	261	228	206
Age	29	30	31	32	33	34	35	36
No. of persons in service	190	181	173	167	161	155	159	146
Age	37	38	39	40	41	42	43	44
No. of persons in service	144	136	131	125	119	113	106	99
Age	45	46	47	48	49	50	51	52
No. of persons in service	93	87	80	73	66	59	53	46
Age	53	54	55	56	57	58	59	60
No. of persons in service	39	33	27	22	18	14	11	0

9. Four jobs are to be processed on a machine as per data listed in the table.

Jobs	Processing Time (Days)	Due Dates
1	4	6
2	7	9
3	2	19
4	8	17

Using shortest processing time rule calculate total tardiness.

10. Find Missing "?" entries.

Customer	Inter arrival time	Arrival time	Service time	Time service begins	Waiting time in queue	Time service end	Time customer spend in system	Idle time of server
1	5	5	5	5	0	10	5	?
2	3	8	6	10	2	16	8	?
3	4	12	4	16	4	20	8	?
4	2	14	3	20	6	23	9	?
5	3	17	1	23	6	24	7	?
			?		?		?	

11. For a bakery, the simulation calculations for a period of 4 - days are given in the following table.

Days	Random number	Demand
1	30	20
2	99	45
3	47	51
4	50	40

What is the demand for random number 30 ?

(A) 45 (B) 20

(C) 51 (D) 40

12. While assigning random numbers in Monte Carlo simulation it is

(A) not necessary to assign the exact range of random number interval as the probability.

(B) necessary to develop a cumulative probability distribution.

(C) necessary to assign the particular appropriate random numbers.

(D) All of the above

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AP-102

April-2023

M.Sc., Sem.-IV

510 : Mathematics

(EB : Functional Analysis – II)

Time : 2:30 Hours] **[Max. Marks : 70**

1. (A) If T is a positive operator on H, then prove that $I + T$ is non-singular. **7**

(B) Let H be a Hilbert space and T_1 and T_2 be self-adjoint, then prove that $T_1 T_2 = 0 \Leftrightarrow R(T_1)$ is orthogonal to $R(T_2)$. **7**

OR

(A) Show that the adjoint operation $T \rightarrow T^*$ on $\beta(H)$ has the following properties :

(i) $(T_1 T_2)^* = T_2^* T_1^*$

(ii) $\| T^* T \| = \| T \|.^2$ **7**

(B) Show that $T \in \beta(H)$ is unitary if and only if T is an isometric isomorphism of H onto itself. **7**

2. (A) State (without proof) finite dimensional spectral theorem. Show that $T \in \beta(H)$ is normal $\Leftrightarrow$ its adjoint T^* is a polynomial in T. **7**

(B) If P is a projection on M then prove that $x \in M \Leftrightarrow P(x) = x \Leftrightarrow \| P(x) \| = \| x \|$. **7**

OR

(A) Let H be a finite dimensional Hilbert space over $\mathbb{C}$ and let $T \in \beta(H)$. Define the spectrum $\sigma(T)$ of T. Show that it is a non-empty compact subset of $\mathbb{C}$. **7**

(B) If P and Q are projections on closed linear subspaces M and N of H respectively, prove that $P \leq Q \Leftrightarrow M \subseteq N \Leftrightarrow PQ = P$. **7**

3. (A) Let X be a normed linear space and $A \in BL(X)$. Define the spectrum $\sigma(A)$, eigen spectrum $\sigma_e(A)$ and the approximate eigen spectrum $\sigma_a(A)$ of A. Show that $\sigma_e(A) \subseteq \sigma_a(A) \subseteq \sigma(A)$. **7**

(B) Find the spectrum of right shift operator on l^2. **7**

OR

(A) State and prove Gelfand-Mazur theorem. **7**

(B) Find the spectrum of $A(x) = \left(x_1, \frac{x_2}{2}, \frac{x_3}{3}, \ldots\right)$, where $x \in l^p$. **7**

4. (A) Prove that $F \in BL(X, Y)$ is compact if and only if for every bounded sequence (x_n) in X, $(F(x_n))$ has a subsequence which converges in Y. **7**

(B) Prove that any linear map T from $\mathbb{R}^n \to \mathbb{R}^m$ is compact. **7**

OR

(A) Prove that the space $CL(X, Y)$ of all compact linear maps from X to Y is a linear subspace of the space $BL(X, Y)$ of bounded linear maps. **7**

(B) Give an example of a bounded linear operator that is not compact. **7**

5. Attempt any **SEVEN** of the following : **14**

(1) What is the dimension of $\beta(\mathbb{R}^n)$?

(A) n (B) n^n

(C) n^2 (D) 2^n

(2) If $P \in \beta(H)$ is a projection then which of the following statements are true ?

(A) P is positive (B) P is idempotent

(C) P is normal (D) P is isometry

(3) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be given by $T(x, y) = (2y, 3x)$. Then T is _______.

(A) self-adjoint (B) unitary

(C) normal (D) none of these

(4) Let G denote the set of all invertible matrices in algebra $M_{2\times 2}(\mathbb{R})$. Then _______.

(A) G is closed (B) G is connected

(C) G is open (D) G is disconnected

(5) Let $T \in \beta(H)$ be isometry. Then _______.

(A) T is one-one (B) T is invertible

(C) T is onto (D) None of these

(6) Let H be finite dimensional and $T \in \beta(H)$ such that $T^3 = 0$. Then

(A) $\sigma(T) = \{0\}$

(B) $\sigma(T)$ is empty

(C) $\sigma(T)$ may contain a non-zero scalar

(D) none of these

(7) Can we have an operator T on H such that the spectrum of T, $\sigma(T) = \mathbb{C}$?

(A) Yes (B) No

(C) Yes, if H is infinite dimensional (D) Yes, if H is finite dimensional

(8) Let $f \in X = (C[0, 1], \|\cdot\|\infty)$ be defined by $f(x) = x - \frac{1}{2}$, for each $x \in [0, 1]$. What is the spectrum $\sigma(f)$ of f ?

(A) $\sigma(f) = \{0\}$ (B) $\sigma(f) = \left\{\frac{1}{2}\right\}$

(C) $\sigma(f) = \{0, 1\}$ (D) none of these

(9) What is the spectral radius $r_\sigma(T)$ of the linear continuous map $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by $T(x, y, z) = (x, y, 0)$?

(A) 0 (B) 1

(C) 2 (D) 3

(10) Let X be a sequence space which is a Banach space. Let (α_n) be a sequence of scalars converging to 0. For each $x \in X$, let $A(x_1, x_2, x_3, \ldots) = (0, k_1 x_1, k_2 x_2, \ldots)$. Then _______.

(A) $\sigma(A) = \{0\}$

(B) $\sigma(A) = \{\alpha_n / n = 1, 2, \ldots\}$

(C) $\sigma_e(A) = \{0\}$

(D) none of these

(11) Let $T : C[0, 1] \to \mathbb{R}$ be defined by $T(f) = f\left(\frac{1}{2}\right)$, for each $f \in C[0, 1]$. Then

(A) T is a functional

(B) T is not continuous

(C) T is compact

(D) none of these

(12) Let $H = l_2$, the space of all (real) square – summable sequences, and let $S = \{e_1, e_2, e_3, \ldots\}$ be a subset of l_2. Then which of the following statements are true ?

(A) S is closed

(B) S is compact

(C) S is bounded

(D) S contains a convergent subsequence

____