AF-130

April-2023

B.Sc., Sem.-VI

CC-311 : Mathematics

(Convex Analysis and Probability Theory)

Time : 2½ Hours] [**Max. Marks : 70**

Instructions : (i) Notations are usual everywhere.
(ii) Figures to the right indicate marks of the question/Sub-question.

1. (a) Define a convex set and prove that G ∩ H is convex if G and H are convex sets. **9**

    (b) Describe convex hull of a set, giving a suitable example of convex hull of a subset of R^2. **9**

    OR

    (a) Define concave functions on an interval I. **9**

    Also show that the function f : R → R defined as $f(x) = x^3$ is a convex function on $[0, ∞)$ whereas concave on $(−∞, 0]$.

    (b) If I is an interval containing more than one point and f : I → R is a differential function then prove that if f' is non-negative throughout I then f is monotonically increasing on I. **9**

2. (a) Define terms : **9**

    convex set, non-convex sets, affine set.

    Also explain each of them by an example.

    (b) State and Prove the Bivariate Intermediate Value Theorem. **9**

    OR

    (a) Using the addition rule of probability for two events A and B defined on a finite sample space such that P[A] = 0.45, P[B] = 0.55 and P[AUB] = 0.65, then find the probability of following events : (i) $\overline{A}$, (ii) $\overline{A} \cap \overline{B}$, (iii) $\overline{A \cup B}$, (iv) $A \cap \overline{B}$. **9**

    (b) Two balanced dice are thrown once, simultaneously. Find the probability of the following events : **9**

    (i) 2 on a first die and even number on a second die.

    (ii) Odd number on first die and a 3 on second die.

    (iii) Sum of numbers on two dice is 8.

    (iv) Sum of numbers on two dice is divisible by 5.

3. (a) If a random variable X follows binomial distribution with parameters n and p, then, state the value of E(X) and V(X) respectively.

A X follows binomial distribution with parameters n = 27 and p = 1/3, state the value of mean and variance. Also, find $P(X = 0)$, $P(X \leq 2)$. **9**

(b) A random variable X follows Poisson distribution with parameter m, such that $P(X = 2) = P(X = 3)$, then find parameter m and also find $P(X = 0)$, $P(X < 2)$, $P(X > 2)$. **9**

OR

(a) For a normal distribution, state its probability distribution function. Also, state mean, variance, mode and median of normal distribution. **9**

(b) State the probability mass function of Poisson distribution. State its mean and variance. Also, state the situations, where Poisson distribution is applied. If a random variable X has Poisson distribution with mean 2, find $P(X = 1)$, $P(X < 2)$, $P(1 < X < 4)$. **9**

$$\{e^{-1} = 0.368, e^{-2} = 0.135, e^{-3} = 0.050\}$$

4. Attempt any **Eight** of the following questions in short : **16**

(a) Define a Euclidean space.

(b) Define following terms :

(i) Hyper Plane

(ii) Convex combination

(c) Explain convex cone with figure.

(d) State the Intermediate Value Theorem.

(e) Define Convex and concave functions on an interval I.

(f) If $A = \{ (x, y) \in R^2 / x^2 + y^2 \leq 5 \}$, then find the convex hull of A.

(g) Two coins are tossed, find the probability that exactly two heads appear.

(h) If A and B be two events, then, give your comments on following statements.

(i) If $P(A \cap B) = P(A)P(B)$ then, events A and B are independent events.

(j) If $A = \phi$, then value of $P(A \mid B)$ more than 0.

(k) State Bayes' Rule of probability.

(l) For two mutually exclusive events A, B on a finite sample space S,

$P(\overline{A} \mid B) = 1$. Do you agree ? If yes, justify.

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AF-130

April-2023

B.Sc., Sem.-VI

CC-311 : Mathematics

(Operations Research)

Time : 2½ Hours]   **[Max. Marks : 70**

Instructions : (1) **All** the questions are compulsory.
　　　　　　　(2) Notations and terminologies are standard.
　　　　　　　(3) Figure to the right indicates the full marks.

1. (a) Explain Economic Order Quantity (EOQ) model with finite replenishment rate.   **9**

   (b) A company uses rivets at a rate of 5000 kg. per year, rivets costing ₹ 2 per kg. It costs ₹ 20 to place an order and the carrying cost of inventory is 10% per annum. How frequently should order for rivets be placed and how much ?   **9**

OR

   (a) Explain Order-Level, Lot-Size System model. Find minimum cost for this model and optimum shortage level.   **9**

   (b) The demand for an item in a company is 8000 units per year, and the company can produce the item at a rate of 3000 per month. The cost of one set-up is ₹ 500 and the holding cost of one unit per month is ₹ 0.15. The shortest cost of one unit is ₹ 20 per month. Determine the optimum manufacturing quantity and the number of shortages. Also determine the manufacturing time and the time between two set-ups.   **9**

2. (a) Explain basic difference between PERT and CPM.   **9**

   (b) Consider the following data regarding the project.   **9**

Activity	A	B	C	D	E	F	G	H	I
Immediate Predecessor	-	A	B	B	C	D	C	E, F	G, H
Duration	5	7	2	3	1	2	1	3	10

Find the critical path.

OR

   (a) Construct the project network. Find total and free float for each non-critical activity.   **9**

Act.	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
Imm. Pred.	–	–	–	A	A	B	B	C	C	D	E	F	G	H	I	J, K, L	M, N, O
Duration	4	8	5	4	5	7	4	8	3	6	5	4	12	7	10	5	8

   (b) Consider the following project. Draw a network diagram.   **9**

Activity	A	B	C	D	E	F	G	H	I	J	K	L
Predecessors	–	–	–	A	A	E	B	B	D, F	C	H, J	G, I, K

3. (a) Explain dominance principal. **9**
   (b) Solve the following game by simplex method. **9**

Player B

		B₁	B₂	B₃
	A₁	1	−1	−1
Player A	A₂	−1	−1	3
	A₃	−1	2	−1

OR

   (a) Obtain the formula to find optimum strategies in 2×2 two-person zero sum game with strategy. **9**
   (b) Solve the following game : **9**

Player B

		B₁	B₂	B₃	B₄
	A₁	−1	2	3	0
Player A	A₂	−4	−1	−1	0
	A₃	−1	1	1	−4
	A₄	4	−1	2	−7

4. Attempt any **eight** in short : **16**
   (1) List the costs which are involved in inventory problem.
   (2) Define : Lead time, Demand.
   (3) What is the carrying cost in EOQ model with finite replacement rate if production rate = demand rate ?
   (4) Given the following information, develop a network.

Activity	Immediate Predecessor
A	–
B	–
C	A
D	B

   (5) Define : Project.
   (6) Define : Two-person zero sum game.
   (7) Define : Free Float
   (8) Solve the following game.

Player B

		B₁	B₂	B₃	B₄
	A₁	8	−2	9	−3
Player A	A₂	6	5	6	8
	A₃	−2	4	−9	5

   (9) Define : A mixed strategy.
   (10) List any two methods which are used to solve the games without saddle point.
   (11) Define : Free float
   (12) Define : Inventory.

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