Seat No. : ______________

AC-111

April-2023

B.Sc., Sem.-VI

CC-308 : Mathematics
(Analysis – II)

Time : 2:30 Hours] **[Max. Marks : 70**

Instructions : (1) **All** the questions are compulsory.

(2) Notations and Terminology are standard.

(3) Figures to the right indicates the full marks.

1. (a) Define Riemann integrable function. Prove : If f is a monotone function on [a, b] then $f \in R[a, b]$. **7**

(b) Let $f(x) = x^2$ on [0, 1]. For $n \in \mathbb{N}$, define $P_n = \left\{0, \frac{1}{n}, \frac{2}{n}, \frac{3}{n}, ..., \frac{n-1}{n}, 1\right\}$ then compute $\lim\limits_{n\to\infty} U_{P_n}$ and $\lim\limits_{n\to\infty} L_{P_n}$. Is the function integrable ? If so, find the value of the integral. **7**

OR

(a) State and prove First Mean Value Theorem of Integral Calculus. **7**

(b) State Second Mean Value Theorem of Integral Calculus. Find a point c in $\left[0, \frac{\pi}{2}\right]$ such that $\int_0^1 \frac{1}{1+x^2} dx = 1$. **7**

2. (a) If Σa_n diverges for all $a_n > 0$ then show that the series $\Sigma \frac{a_n}{1+na_n}$ is divergent. **7**

(b) State and prove comparison test. Hence check the convergence of $\sum_{n=0}^{\infty} \frac{1}{n!}$. **7**

OR

(a) Define conditional convergence of the series. If $\Sigma\ a_n$ is absolutely convergent series then prove that it is convergent. Is the converse true ? **7**

(b) State condensation test. Hence check the convergence of $\sum_{n=2}^{\infty} \frac{1}{n(\log n)^{\alpha'}}$ $\alpha \in \mathbb{R}$. **7**

3. (a) If Σa_n converges absolutely to A, then prove that any rearrangement of Σa_n also converges to A. **7**

(b) Define Cauchy product of two series. If the series Σa_n and Σb_n converge absolutely to A and B respectively then prove that their Cauchy product series Σc_n is convergent and if C is the sum of Cauchy product then C = AB. **7**

OR

(a) State and prove Mertens' theorem. **7**

(b) Discuss the convergence of following improper integrals : **7**

(1) $\int_1^{\infty} \frac{1}{x^2}\, dx$

(2) $\int_0^1 \frac{1}{x^2 + x^{1/2}}\, dx$

4. (a) State and prove Binomial series theorem. **7**

(b) Derive Taylor's formula with the integral form of the remainder for f(x) = cos x about a = 0 in ($-\infty$, ∞). **7**

OR

(a) For $-1 < x < 1$, prove that $\log(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - ... + (-1)^{n-1}\frac{x^n}{n} + \dots$ (Use Cauchy's form of remainder). **7**

(b) Find a power series solution of $y'' - xy = 0$ with $y(0) = 1$ and $y'(0) = 0$. **7**

5. Attempt any **seven** questions in short : **14**

(a) Give one function which is not Riemann integrable.

(b) Is $\lim_{n\to\infty} x_n = 0$ a sufficient condition for convergence of $\sum_{n=0}^{\infty} x_n$? Justify.

(c) Give example of absolutely convergent series.

(d) Find a power series solution of y' – y = 0.

(e) Does | f | ∈ R[a, b] implies f ∈ R[a, b] ? Justify.

(f) Give example of conditionally convergence series.

(g) Discuss convergence of $\sum_{n=1}^{\infty} \frac{n^2(x-2)^n}{2^n}$.

(h) State Taylor's formula with Lagrange's form of the remainder.

(i) Verify First Mean Value Theorem of Integral Calculus for the function $f(x) = 2x + 1$ on [0, 1].

(j) State Cauchy's root test.

(k) Define improper integral of the second kind.

(l) Define the exponential of x.
