Seat No. : _______________

AB-156

April-2019

M.Sc., Sem.-II

407 : Physics

(Quantum Mechanics-II and Mathematical Physics-II)

Time : 2:30 Hours] **[Max. Marks : 70**

Instructions : (1) Attempt **all** questions.

(2) **All** questions carry equal marks.

(3) Symbols and terminology have their usual meanings.

(4) Scientific calculator may be permitted.

1. (A) (i) Show that Schrödinger approach leads to Hamiltonian-Jacobi equation of motion. **7**

(ii) Discuss Thomas-Fermi approximation. Obtain differential equation $\frac{d^2\chi}{dx^2} = \frac{\chi^{\frac{3}{2}}}{x^{\frac{1}{2}}}$. **7**

OR

(i) How Schrödinger equation for the system of z-electron can be solved ? Discuss in detail. Obtain integro-differential equation and explain how it can be solved.

(ii) Show that expectation value of $\langle \hat{A} \rangle_{\Psi}$ is constant of motion in Schrödinger picture.

(B) Write any **FOUR** out of **SIX :** **4**

(i) The symmetry character of wave function is not changing with time. [True or False]

(ii) If we change two columns of the slater determinant then symmetric wave function remains symmetric. [True or False]

(iii) Consider two system having n-1 and n number of electrons then find out ratio of the normalization constant.

(iv) Show that up spin states and down spin states wave functions are orthogonal to each other.

(v) What will be the value of $[H\ P_{12}]$?

(vi) If angular momentum $\hat{L}$ is the generator of unitary transformation then write $\hat{U}(t, t_0)$.

2. (A) (i) Write Maxwell's equations. By solving Maxwell's equations vector potential $\vec{A}(\vec{r}, t)$ is given by $\vec{A}(\vec{r}, t) = [A_k \exp(-i\omega_k t) \exp(i\vec{k}.\vec{r}) + A_k^* \exp(i\omega_k t) \exp(-i\vec{k}.\vec{r})]$ for k^{th} mode of vibration. Find out average energy of electromagnetic radiation E_k for k^{th} mode of vibrations in terms of A_k and A_k^*. **7**

(ii) Show that coherent states are identical to classical states. Find out average $\langle \Delta p \rangle_\alpha^2$ in the coherent states α. Take $(\Delta q^2) = \frac{\hbar\omega}{2}$. **7**

OR

(i) Calculate A_{21} probability of spontaneous emission in H-atom when atom is coming from one of the 2p level to 1s level.

(ii) Find out transition probability $C_m(t)$ when transition is taking place from $|m\rangle$ to $|n\rangle$ states.

(B) Write any **FOUR** out of **SIX** : **4**

(i) Find out average value of $\langle \sec\theta \rangle$ over a solid angle.

(ii) Show that ratio of two fock's states $|n + 1\rangle$ and $|n\rangle$ is $\frac{(n!)^{\frac{1}{2}}}{(n+1)!^{\frac{1}{2}}} \hat{a}^+$.

(iii) What will be expectation value of $\hat{a}a^+$ in the coherent state ?

(iv) What will be the unit of Einstein coefficient B_{12} ?

(v) If $\vec{A} = 3x^2 - 4yz$ then find out B_x.

(vi) What is the unit of phase factor $(\omega t - \vec{k}.\vec{r})$?

3. (A) (i) Prove Cauchy's integral theorem: If function f(Z) is analytic at all points within a simply connected region, and C is piecewise smooth then, $\oint_c f(z)dZ = 0$. **7**

(ii) (a) Find out Residue R(l) of function $\frac{e^z}{z-1}$. **4**

(b) Find out Residue $R\left(-\frac{1}{2}\right)$ and R(5) of function $f(Z) = \frac{z}{(2z+1)(5-z)}$ **3**

OR

(i) Prove the Residue theorem : $\oint_c f(Z)\, dZ = 2\pi i\,(R_1 + R_2 + ...+R_n)$. **7**

(ii) Show that $f(Z) = f(Z_0) + f'(Z_0)(Z - Z_0) + ... \frac{f^n}{n!}(Z_0)(Z - Z_0)^n$ for $|(Z - Z_0)| < r$, where radius of a circle C_0 and f(Z) is analytic everywhere inside C_o centered at Z_0. **7**

(B) Write any **THREE** out of **FIVE** : **3**

(i) If $Z_1 = a_1 + ib_1$ and $Z_2 = a_2 + ib_2$ then $Z_1 + Z_2$ = ________, find out real and imaginary parts of the sum.

(ii) Complex conjugate of the complex number Z is given by ____ .

(iii) What do you understand by an analytic function ?

(iv) If $F(Z) = Z^2$ then find out $u(x, y)$ and $v(x, y)$.

(v) If $F(Z) = |Z|^2$ then find out real and imaginary parts of it.

4. (A) (i) Write a general form of an integral equation. Convert a 2nd order differential equation into an integral equation. **7**

(ii) Explain separable Kernel method for solving an integral equation. **7**

OR

(i) Explain how Green's function is found useful to solve one dimensional problem ? Discuss how is the homogenous differential equation with non-homogeneous boundary conditions transferred into non-homogeneous equation with homogeneous boundary conditions.

(ii) Construct the Green's function for $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + (k^2x^2 - 1)y = 0$, subject to the boundary conditions: $y(0) = 0$ and $y(1) = 0$.

(B) Write any **THREE** out of **FIVE** : **3**

(i) How can we define unknown function $\varphi(x)$ in Neumann's method ?

(ii) Write an expression for Volterra equation of the 1st kind.

(iii) What do you mean by first kind and second kind integrals ?

(iv) State mathematical expression for Fredholm equation for 2nd kind.

(v) Write the expression for self-adjoint differential equation.