Seat No. : _____

AB-160

April-2019

M.Sc., Sem.-II

407 : Statistics
(Reliability and Life Testing)
(New Course)

Time : 2:30 Hours] **[Max. Marks : 70**

Instructions : (1) Scientific calculator is allowed to use.
(2) Statistical table will be given on demand.

1. (A) Answer the following :

(i) Define hazard function h(t) of a continuous random variable X. Show that **7**

$$\frac{P(X > b)}{P(X > a)} = \exp\left\{-\int_a^b h(t)dt\right\}$$

(ii) In usual notations prove that $f(t) = h(t) \exp\left(-\int_0^t h(x)dx\right)$. Hence deduce $f(x)$ for $h(t) = \frac{\beta}{\theta}\left(\frac{x-\mu}{\theta}\right)^{\beta-1}$, $x > \mu$. **7**

OR

(i) Let $X_1, X_2, ..., X_n$ be iid exponential life times of n items. Obtain the distribution of the time between the two consecutive ordered failures. State its mean and variance.

(ii) Let random variable X has exponential life time distribution then prove that

$$E(X - c \backslash X > c) = E(X)$$

(B) Attempt any **four** : **4**

(i) Define reliability.

(ii) Define mtbf.

(iii) Define mttf.

(iv) State the life time model with constant failure rate.

(v) State the formula to estimate R(t) for ungrouped data based on non-parametric method.

(vi) Define cumulative hazard function.

2. (A) Answer the following :

(i) Distinguish between Type-I and Type- II censoring. Construct general form of the likelihood functions in case of type-II censoring under WOR and WR. **7**

(ii) Under type-II censoring WOR obtain mle of mean life 0 for exponential life time model. Hence construct 95% confidence interval for θ. **7**

OR

(i) Under Type-I censoring WOR obtain mle of θ for life time model

$$f(x, \theta) = \frac{x}{\theta^2} e^{\frac{-x^2}{2\theta^2}}, \; x > 0, \; \theta > 0.$$ Obtain its asymptotic variance.

(ii) Under type-II censoring WOR obtain mle of the parameters of the Weibull life time model. Suggest the method to obtain asymptotic variance of the estimators.

(B) Attempt any **four** : **4**

(i) What is censoring ?

(ii) State TTT under type-II censoring WOR for 2-parameter exponential life time model.

(iii) State the relation between expected test termination time under WOR and WR in case of exponential life time model.

(iv) State the distribution of number of failures in type-I censoring WOR.

(v) State the distribution of number of failures in type-I censoring WR.

(vi) State the distribution of TTT under type-II censoring WOR for exponential life time model.

3. (A) Answer the following :

(i) Obtain UMVUE of mtbf and R(t) in case of exponential life time model for type-II censoring WOR. **7**

(ii) Determine necessary sample size with minimum total cost under type-II censoring WOR with necessary assumptions. **7**

OR

(i) Obtain UMVUE of mtbf and R(t) in case of exponential life time model for type-II censoring WR.

(iii) Discuss Fisher's method for estimating the parameters of Weibull life time model based on failure rate estimate.

(B) Attempt any **three** : **3**

(i) State UMVUE of hazard rate for exponential life time model with mean θ under type-II censoring WOR.

(ii) State UMVUE of expected termination time for exponential life time model with mean θ under type-II censoring WR.

(iii) State the formula of necessary sample size under type-II censoring WR for exponential life time model with mean θ.

(iv) State UMVUE of μ under type-II censoring WOR for 2-parameter exponential life time model with $X > \mu$, and hazard rate $1/\theta$.

(v) State the estimate of hazard rate given by Sinha and Fu for Weibull life time model.

4. (A) Answer the following :

(i) In case of a series system with n components and i-th component with cdf $F_i(x)$ and hazard rate hi(x), i = 1, 2, ..., n show that the pdf of life time of the series system is given by $f_s(t) = \Pi_{i=1}^{n}[1 - F_i(t) \Sigma_{i=1}^{n} h_i(t)$. **7**

(ii) Obtain reliability of Series-Parallel system and Parallel-Series systems. Which of the two systems do you prefer ? Why ? **7**

OR

(i) Let the life time of the two independent components are exponential with hazard rate $1/\theta_1$ and $1/\theta_2$, show that the expected life of series system with such two components is less than that of either component but in case of parallel system it is reversed.

(ii) A system consists of four iid components in parallel configuration. What must be the reliability of each component if the overall reliability of the system is R ? Calculate it in case of series system also.

(B) Attempt any **three** : **3**

(i) State reliability of a series system.

(ii) State reliability of parallel system.

(iii) State reliability of 3 out of 4 system with identical components.

(iv) If components are very much unreliable then which of the two systems series and parallel do you prefer ?

(v) State the structure function of parallel system.

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AB-160

April-2019

M.Sc., Sem.-II

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(Reliability Literature and Bayes Estimation)
(Old Course)

Time : 2:30 Hours] **[Max. Marks : 70**

Instructions : (1) Scientific calculator is allowed to use.
(2) Statistical table will be given on demand.

1. (A) Answer the following :

(i) In case of a series system with n components and i-th component with cdf $F_i(x)$ and hazard rate hi(x), i = 1, 2, ..., n show that the pdf of life time of the series system is given by $f_s(t) = \Pi_{i=1}^{n}[1 - F_i(t) \sum_{i=1}^{n} h_i(t)$. **7**

(ii) Obtain reliability of Series-Parallel system and Parallel-Series systems. Which of the two systems do you prefer ? Why ? **7**

OR

(i) Let the life time of the two independent components are exponential with hazard rate $1/\theta_1$ and $1/\theta_2$, show that the expected life of series system with such two components is less than that of either component but in case of parallel system it is reversed.

(ii) A system consists of four iid components in parallel configuration. What must be the reliability of each component if the overall reliability of the system is R ? Calculate it in case of series system also.

(B) Attempt any **four** : **4**

(i) State reliability of a series system.

(ii) State reliability of parallel system.

(iii) State reliability of 3 out of 4 system with identical components.

(iv) If components are very much unreliable then which of the two systems series and parallel do you prefer ?

(v) State the structure function of parallel system.

(vi) State the structure function of series system.

2. (A) Answer the following :

(i) Define hazard function h(t) of a continuous random variable X. Show that **7**

$$\frac{P(X > b)}{P(X > a)} = \exp\left\{-\int_a^b h(t)dt\right\}$$

(ii) In usual notations prove that $f(t) = h(t) \exp\left(-\int_0^t h(x)dx\right)$. Hence deduce $f(x)$ for $h(t) = \frac{\beta}{\theta}\left(\frac{x-\mu}{\theta}\right)^{\beta-1}$, $x > \mu$. **7**

OR

(i) Let $X_1, X_2, ..., X_n$ be iid exponential life times of n items. Obtain the distribution of the time between the two consecutive ordered failures. State its mean and variance.

(ii) Let random variable X has exponential life time distribution then prove that

$$E(X - c \backslash X > c) = E(X)$$

(B) Attempt any **four** : **4**

(i) Define reliability.

(ii) Define mtbf.

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(iv) State the life time model with constant failure rate.

(v) State the formula to estimate R(t) for ungrouped data based on non-parametric method.

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3. (A) Answer the following :

(i) Distinguish between Type-I and Type- II censoring. Construct general form of the likelihood functions in case of type-II censoring under WOR and WR. **7**

(ii) Under type-II censoring WOR obtain mle of mean life θ for exponential life time model. Hence construct 95% confidence interval for θ. **7**

OR

(i) Under Type-I censoring WOR obtain mle of θ for life time model

$f(x, \theta) = \frac{x}{\theta^2} e^{\frac{-x^2}{2\theta^2}}$, $x > 0$, $\theta > 0$. Obtain its asymptotic variance.

(ii) Under type-II censoring WOR obtain mle of the parameters of the Weibull life time model. Suggest the method to obtain asymptotic variance of the estimators.

(B) Attempt any **three** : **3**

(i) What is censoring ?

(ii) State TTT under type-II censoring WOR for 2-parameter exponential life time model.

(iii) State the relation between expected test termination time under WOR and WR in case of exponential life time model.

(iv) State the distribution of number of failures in type-I censoring WOR.

(v) State the distribution of number of failures in type-I censoring WR.

4. (A) Answer the following :

(i) Under squared error loss function obtain the general form of the Bayes estimator of $g(\theta)$, a function of parameter θ. **7**

(ii) Let $X_1, X_2, ..., X_n$ be a random sample from Poisson distribution with mean θ, $\theta > 0$. Let prior distribution for θ exponential with mean $\lambda > 0$. Obtain Bayes estimator of θ and θ^2 under squared error loss function. **7**

OR

(i) Define Bayes estimator. Obtain extensive rule to obtain Bayes estimator.

(ii) Let $X_1, X_2, ..., X_n$ be a random sample from Bernoulli distribution with mean θ, $0 < \theta < 1$. Let prior distribution for θ uniform U(0, 1). Obtain Bayes estimator of $1/\theta$ and $1/\theta^2$ under squared error loss function.

(B) Attempt any **three** : **3**

(i) Define prior distribution.

(ii) Define posterior distribution.

(iii) Define risk function.

(iv) State weighted squared error loss function.

(v) State the most appropriate prior distribution for the parameter θ, of Rayleigh distribution.
