Seat No. : _______________

AC-166

April-2019

M.Sc., Sem.-II

408 : Statistics
(Distribution Theory)
(New Course)

Time : 2:30 Hours] **[Max. Marks : 70**

1. (A) Write the following.

(i) Define Contagious Distribution. Write applications of Contagious Distribution. Let $X_1, X_2, \dots, X_N$ are independent discrete random variables and N is also a random variable independent of X_i's. If $Y = \sum_{i=i}^{n} x_i$ and ϕ_i, i = 1, 2, 3 are the characteristic functions of random variables N, X and Y respectively then Obtain Characteristic function ϕ_3 as a compound function of ϕ_1 and ϕ_2. **7**

(ii) Define Neyman type A distribution. Estimate the parameters of this distribution using method of moments. Also obtain recurrence relation of probability for this distribution. **7**

OR

(i) Define Poisson-Binomial distribution. Obtain its probability generating function. Stating necessary assumptions, show that Poisson-Binomial distribution tends to Poisson-Poisson Distribution. **7**

(ii) Define Poisson-Pascal distribution. Obtain recurrence relations for Probabilities and descending factorial moments for Poisson-Pascal distribution. **7**

(B) Answer the following questions. (Any **four**) **4**

(i) Write the probability mass function of the Poisson-Binomial distribution.

(ii) Write the probability mass function of the Poisson Negative Binomial distribution.

(iii) Write the probability generating function of the Poisson Negative Binomial distribution.

(iv) Write the recurrence relation for the probability of Neyman type-B distribution.

(v) Write the probability mass function of the Poisson-Poisson distribution.

(vi) Write any two applications of Neyman type A Distribution.

2. (A) Write the following.

(i) Discuss the roll of non-central distributions in statistical inference. If $X \sim N(\mu, 1)$ then, obtain pdf of non-central Chi-square distribution using M.G.F. **7**

(ii) Define non-central 'F' distribution with (n_1, n_2) degrees of Freedom. In usual notations, obtain probability density function of non-central 'F' distribution. **7**

OR

(i) Define non-central 't' statistics. In usual notations obtain probability density function of non-central 't' distribution.

(ii) State and prove the relation between non-central chi-square, non-central F and non-central 't' distributions. **7**

(B) Answer the following questions. (Any **four**) **4**

(i) If a random variable X has a chi-square distribution with d.f. 'r' and a random variable Y has a non-central chi-square distribution with d. f. 1 and non-centrality parameter λ then write the distribution of the random variable $Z = X + Y$.

(ii) A non-central chi-square distribution is a Compound distribution of which two distributions ?

(iii) Write the moment generating function of non-central chi-square distribution.

(iv) If X is a non-central chi-square variate with d. f. 5 and non-centrality parameter $\delta = 2$ then obtain E(X) and V(X).

(v) If a statistics t follows Student's t distribution with d.f. n, then write the distribution of t^2.

(vi) If $X_1, X_2, ..., X_n$ are n independent random variables each distributed as $N(\mu, 1)$ then what is the distribution function of random variable $w = X_1 / \left(\frac{1}{(n-1)} \sum_{i=2}^{n} X i^2 \right)^{1/2}$

3. (A) Write the following.

(i) Define the sample range. Obtain the distribution of sample range for infinite range population. If a random sample of size n is taken from exponential distribution with mean 1/3, what is the probability that the sample range does not exceed 2 ? **7**

(ii) Obtain the distribution of sample range for finite range population. If X has uniform distribution $U(0, \theta)$, then show that $E(R) = [(n-l)/(n+l)]\,\theta$ based on a random sample of size n taken from the given distribution, where R = sample range. **7**

OR

(i) Obtain the distribution of sample median when n is even as well as odd number. **7**

(ii) If $X_{(n)} = \max\{X_1, X_2, ... , X_n\}$ then show that

$$E(X_{(n)}) = E(X_{(n-1)}) + \int_0^{\infty} F^{(n-1)}(x)(1 - F(x))\,dx$$

Also find $E(X_{(r)})$ for $F(x) = 1 - e^{-\theta x}; \theta > 0, x \geq 0.$ **7**

(B) Answer the following questions. (Any **three**) **3**

(i) Define ordered statistics.

(ii) Write any two applications of ordered statistics.

(iii) Write the distribution of smallest ordered statistic.

(iv) If a random sample of size 7 is taken from Uniform distribution then write the probability density function of the sample median.

(v) Write mean and variance of rth ordered statistic for U (0,1) distribution.

4. (A) Write the following.

(i) Prove that $(n - r)\ \mu_{r:n}^{(k)} + r\ \mu_{r+1:n}^{(k)} = n\mu_{r:n-1}^{(k)}$ where $\mu_{r:n}^{(k)}$ denotes k^{th} row moment of r^{th} order statistic from a random sample of size n. **7**

(ii) Explain the procedure to obtain Confidence Interval for p^{th} Quantile of the distribution. If $X_{(r)}$ be the r^{th} order statistic of a random sample of size 7 taken from any continuous distribution with cumulative distribution function $F_x(x)$ then obtain

$p\,(X_{(3)} < \text{Population median} < X_{(5)})$ **7**

OR

(i) In usual notations obtain the formula for correlation coefficient between the rank-orders and variate values. **7**

(ii) Write a brief note on Sign Statistic. **7**

(B) Answer the following questions. (Any **three**) **3**

(i) Define rank-order statistics with appropriate example.

(ii) Give functional definition of rank-order statistics.

(iii) Write an application of rank order statistics.

(iv) If X has exponential distribution with mean θ, then write the approximate expressions for $E(X_{(5)})$ for a sample of size 9.

(v) What is the difference between Sign Statistic and Wilcoxon signed rank Statistic ?

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1. (A) (i) Define Neyman type A distribution. Obtain its probability generating function. Hence derive its r^{th} factorial cumulant. Also describe the method of fitting Neyman type A distribution to the numerical data. **7**

(ii) Let $x_1, x_2, ..., x_N$ are N identically independently distributed random variables and N is also a random variable independent of x_i 's . Show that

(i) $E(S_N) = E(N) E(X)$

(ii) $V(S_N) = E(N) V(X) + V(N) \{E(X)\}^2, S_N = \sum_{i=1}^{N} X_i$. **7**

OR

(i) Describe the method of maximum likelihood to estimate the parameters of the Poisson Poisson distribution.

(ii) Define Poisson - Binomial distribution. Obtain its probability generating function. Show that Poisson - Binomial distribution tends to Poisson - Poisson distribution. State necessary assumptions involved.

(B) Answer the following questions : (Any **four**) **4**

(i) Write the probability mass function of the Poisson Bionomial distribution,

(ii) Write the probability mass function of the Poisson Negative Binomial distribution.

(iii) Write the probability generating function of the Poisson Negative Binomial distribution.

(iv) Write the recurrence relation for the probability of Neyman type-A distribution.

(v) When Neyman type-B distribution tends to Neyman type-A distribution ?

(vi) Write any two applications of Contagious Distribution.

2. (A) (i) Define Poisson - Pascal distribution. Obtain recurrence relations for Probabilities and descending factorial moments for this distribution. **7**

(ii) Discuss the roll of non-central distributions in statistical inference. If X~ N(μ, l) then, obtain pdf of non-central Chi-square distribution using M.G.F.. **7**

OR

(i) Define non-central 'F' distribution with (n_1, n_2) degrees of Freedom. In usual notations, obtain probability density function of non-central 'F' distribution.

(ii) Define non-central 't' statistic. In usual notations obtain probability density function of non-central 't' distribution.

(B) Answer the following questions. (Any **four**) **4**

(i) Define Descending factorial cumulant generating function H(t).

(ii) If a random variable X has a chi-square distribution with d.f. 'r' and a random variable Y has a non-central chi-square distribution with d.f. 1 and non-centrality parameter λ then write the distribution of the random variable Z = X + Y.

(iii) Write the moment generating function of non-central chi-square distribution.

(iv) When 'v=l', student's t distribution tends to which distribution ?

(v) If X is a non-central chi-square variate with d. f. 5 and non-centrality parameter δ is also 5 then obtain E(X) and V(X).

(vi) If a statistics t follows Student's t distribution with d.f., then write the distribution of t^2.

3. (A) (i) Obtain the joint probability density function of the largest and the smallest order Statistics. **7**

(ii) Let a random variable 'X' follows an Exponential distribution with mean θ , $\theta > 0$. If a randon sample of size n is taken from this distribution then show that $X_{(r)}$ and $X_{(s)} - X_{(r)}$ are independently distributed. **7**

OR

(i) Define the sample range. Obtain the distribution of sample range for infinite range population. State the distribution of sample range for finite range population.

(ii) Obtain the distribution of sample median when (i) n is odd number and (ii) n is even number.

(B) Answer the following questions. (Any **three**) **3**

(i) Define ordered statistics.

(ii) Write any two applications of ordered statistics.

(iii) Write the distribution of smallest ordered statistic.

(iv) If a random sample of size 5 is taken from Uniform distribution then obtain the probability density function of the sample median.

(v) Write mean and variance of rth ordered statistic for U (0, 1) distribution.

4. (A) Write the following.

(i) Prove that $(n - r)\ \mu_{r:n}^{(k)} + r\ \mu_{r+1:n}^{(k)} = n\mu_{r:n-1}^{(k)}$ where $\mu_{r:n}^{(k)}$ denotes k^{th} row moment of r^{th} order statistic from a random sample of size n. **7**

(ii) Explain the procedure to obtain Confidence Interval for p^{th} Quantile of the distribution. If $X_{(r)}$ be the r^{th} order statistic of a random sample of size 7 taken from any continuous distribution with cumulative distribution function $F_x(x)$ then obtain

$p\,(X_{(3)} < \text{Population median} < X_{(5)})$ **7**

OR

(i) Define rank-order statistics with appropriate example. Give functional definition of rank-order statistics. In usual notations obtain the formula for the correlation coefficient between the rank-orders and variate values.

(ii) Obtain the correlation coefficient between r^{th} and s^{th} order statistics for the uniform distribution U(0,l). Hence write the correlation coefficient between the smallest and the largest order statistics.

(B) Answer the following questions. (Any **three**) **3**

(i) Define rank-order statistics with appropriate example.

(ii) Give functional definition of rank-order statistics.

(iii) Write an application of rank order statistics.

(iv) If X has exponential distribution with mean θ, then write the approximate expressions for $E(X_{(3)})$ for a sample of size 5.

(v) What is the difference between Ordered Statistic and rank order ?