Seat No. : _______________

AE-135

April-2019

M.Sc., Sem.-II

410 : Mathematics
(New)

Time : 2:30 Hours] **[Max. Marks : 70**

1. (A) Attempt the following : **14**

(i) Find the complete integral of $2zx - px^2 - 2qxy + pq = 0$

(ii) Find the general integral of $(z^2 - 2yz - y^2)p + x(y + z)q = x(y - z)$

OR

(i) Eliminate the arbitrary function f and g from $y = f(x - at) + g(x + at)$ and find the corresponding partial differential equation. If $\vec{X}.\ \text{curl}\vec{X} = 0$ where $\vec{X} = (P, Q, R)$ and μ is an arbitrary differentiate function of x, y and z then prove that $\mu\vec{X}.\ \text{curl}(\mu\vec{X}) = 0$

(ii) Show that the equations $xp = yq$, $z(xp + yq) = 2xy$ are compatible and solve them.

(B) Choose the correct alternative : (any **four**) **4**

(i) Find the corresponding PDE by eliminating the arbitrary function F from the equation $z = F\left(\frac{x}{y}\right)$.

(a) $px^2 + qy^2 = 0$ (b) $px + qy = 0$

(c) $px + 2qy = 0$ (d) $2px + 3qy = 0$

(ii) Find the corresponding PDE by eliminating the parameters 'a' and 'b' from the equation $z = ax + by$.

(a) $z = px$ (b) $z = px + 2qy$

(c) $z = px + qy$ (d) $z = qy$

(iii) The solution of $p + q = z$ is

(a) $f(x + y, y + \log z) = 0$ (b) $f(xy, y\log z) = 0$

(c) $f(x - y, y - \log z) = 0$ (d) None of these

(iv) The equation $\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = 4[(x - 2)^2 + (y - 3)^2]$ is of order _________ and degree _________.

(a) 1, 2 (b) 2, 1 (c) 1, 1 (d) 1, 3

(v) The differential equation $z_x + (x + y)z_y = xy$ is,

(a) Semilinear (b) Quasilinear (c) Linear (d) Non-linear

(vi) The differential equation $xz_x^2 + yz_y^2 = 2$ is,

(a) Semilinear (b) Quasilinear (c) Linear (d) Non-linear

2. (A) Attempt the following : **14**

(i) Find the complete integral of $px y + pq + qy = yz$

(ii) Solve the partial differential equation $z_x z_y = z$ subject to the condition $z(s, -s) = 1$

OR

(i) Find the complete integral of $2p_1 x_1 x_3 + 3p_2 x_3^2 + p_2^2 p_3 = 0$

(ii) Solve the initial value problem for the Quasi-linear equation $z_y + z.\ z_x = 0$ containing the initial data curve C: $z(x, 0) = f(x)$

(B) Choose the correct alternative : (any **four**) **4**

(i) Complete integral of the equation pq = 1 is,

(a) $a^2x + y - az = c$ (b) $x + y - az = c$

(c) $a^2x + y - az = 0$ (d) $a^2x - y - az = c$

(ii) Complete integral of $(p + q)(z - xp - yq) = 1$ is,

(a) $z = ax + by + (a + b)$ (b) $z = ax + by + \frac{1}{(a+b)}$

(c) $z = ax - by - (a + b)$ (d) $z = ax - by - \frac{1}{(a+b)}$

(iii) A solution which contains a number of arbitrary constants equal to the independent variables is called,

(a) Complete integral (b) Particular integral

(c) General integral (d) None of these

(iv) A quasi-linear partial differential equation is represented as,

(a) $Pp + Qq = R$ (b) $P + Q = R$

(c) $Pp - Qq = R$ (d) None of above

(v) Which of the following is not an example of a first order differential equation of Clairaut's form ?

(a) $px + qy - 2\sqrt{pq}$ (b) $px + qy = p^2q^2$

(c) $p^2 + q^2 = z^2(x + y)$ (d) $px + qy + \frac{1}{p-q}$

(vi) If the equation is of the form f(p, q) = 0 then Charpit's equation takes the form,

(a) $\frac{dx}{f_p} = \frac{dy}{f_q}$

(b) $\frac{dx}{f_p} = \frac{dy}{f_q} = \frac{dz}{pf_p + qf_q} = \frac{dp}{0} = \frac{dq}{0}$

(c) $\frac{dx}{f_p} = \frac{dz}{pf_p + qf_q} = \frac{dp}{-(f_x + pf_z)} = \frac{dq}{-(f_y + qf_z)}$

(d) $\frac{dx}{f_p} = \frac{dy}{f_q} = \frac{dp}{-(f_x + pf_z)} = \frac{dq}{-(f_y + qf_z)}$

3. (A) Attempt the following : **14**

(i) Solve the following IVP by Fourier transform method.

PDE: $u_t(x, t) = \alpha^2 u_{xx}(x, t), -\infty < x < \infty, t > 0,$

IC: $u(x, 0) = f(x), -\infty < x < \infty,$

With $u(x, t), u_x(x, t) \to 0$ as $x \to \pm\infty, t > 0.$

(ii) State and solve the heat conduction problem for a finite rod of length l with initial temperature distribution in the rod at time t=0 given by $f(x)$. Use the method of separation of variables.

OR

(i) Reduce the equation $u_{xx} = x^2 u_{yy}$ to a canonical form.

(ii) State the problem of the wave equation in the case of vibrations of a string of finite length and solve it using the method of separation of variables. Consider that both ends are fixed and initial displacement distribution is $f(x)$ and initial velocity distribution is $g(x)$

(B) Choose the correct alternative : (any **three**) **3**

(i) The PDE $u_{tt} - u_{xx} = 0$ is of the type

(a) Parabolic (b) Hyperbolic

(c) Elliptic (d) None

(ii) The PDE $u_{xx} + u_{yy} = 0$ is of the type

(a) Parabolic (b) Hyperbolic

(c) Elliptic (d) None

(iii) If $u_t(x, t) = \alpha^2 u_{xx}(x, t)$ then applying Fourier transform we get

(a) $\frac{d}{dt}\hat{u}(\omega, t) = \alpha^2\omega^2\hat{u}(\omega, t)$ (b) $\frac{d}{dt}\hat{u}(\omega, t) = -\alpha^2\omega^2\hat{u}(\omega, t)$

(c) $\frac{d}{d\omega}\hat{u}(\omega, t) = \alpha^2\omega^2\hat{u}(\omega, t)$ (d) $\frac{d}{d\omega}\hat{u}(\omega, t) = -\alpha^2\omega^2\hat{u}(\omega, t)$

(iv) The PDE $u_{xx} + xu_{yy} = 0, x \neq 0$ is elliptic for,

(a) $x = 0$ (b) $x < 0$

(c) $x = 2$ (d) $x > 0$

(v) For the wave equation the Boundary condition $u(0, t) = 0$ and $u_x(0, t) = 0$ specifies the type

(a) Dirichlet (b) Neumann

(c) Robin (d) Churchill

4. (A) Attempt the following : **14**

(i) Solve the Dirichlet Boundary value problem:

$$u_{xx} + u_{yy} = 0, 0 < x < a, 0 < y < b$$

to the boundary condition

$$u(x, 0) = f(x), \quad 0 \le x \le a$$
$$u(x, b) = 0, \quad 0 \le x \le a$$
$$u(0, y) = 0, \quad 0 \le y \le b$$
$$u(a, y) = 0, \quad 0 \le y \le b$$

(ii) Write the statement of Neumann's Problem for a Circle and solve it.

OR

(i) Show that the exterior Dirichlet problem

$$u_{rr} + \frac{u_r}{r} + \frac{u_{\theta\theta}}{r^2} = 0, \quad 0 \le r < \infty$$

$$u(l, \theta) = 1 + \sin\theta + \cos 3\theta, \ 0 < \theta < 2\pi,$$

has the solution $u(r, \theta) = 1 + \frac{1}{r}\sin\theta + \frac{1}{r^2}\cos 3\theta$.

(ii) If u_1 & u_2 are solutions of Neumann's BVP then show that, $u_1 - u_2$ = constant. Also show that the solution of Neumann's problem is unique upto the addition of a constant.

(B) Choose the correct alternative : (any **three**) **3**

(i) Which of the following is not a solution of the Laplace's equation ?

(a) $f(x, y) = x^2 - y^2$

(b) $f(x, y) = 2xy$

(c) $f(x, y) = ax^2y$, where a = constant

(d) $f(x, y) = 10a$, where a = constant

(ii) Which of the following is a harmonic function ?

(a) $f(x, y) = (x + iy)^5 x$ (b) $f(x, y) = 4(x + iy)^3$

(c) $f(x, y) = (x + iy)^5 y$ (d) $f(x, y) = (x + iy)^3 xy$

(iii) If u is a solution of Neumann's problem for an upper half of plane, then which of the following is true ?

(a) $\int_{-\infty}^{\infty} u_y(x, 0)dx = 5$ (b) $\int_{-\infty}^{\infty} u_y(x, 0)dx = 0$

(c) $\int_{-\infty}^{\infty} u_y(x, 0)dx = x$ (d) $\int_{-\infty}^{\infty} u_y(x, 0)dx = y$

(iv) A boundary condition which specifies value of normal derivative of function is a

(a) Neumann boundary condition

(b) Dirichlet boundary condition

(c) Robin boundary condition

(d) Cauchy boundary condition

(v) If f is continuous function prescribed on the boundary S of a finite simply connected region V, determine a function $\varphi(x, y, z)$ which satisfies $\nabla^2 = 0$ outside V and is such that $\varphi = f$ on S. This type of problem is called

(a) Interior Dirichlet problem (b) Exterior Dirichlet problem

(c) Annulus Dirichlet problem (d) Exterior Neumann problem