Seat No. : _______________

AA-151

April-2019

F.Y. Integrated M.Sc., (C.A. & I.T.), Sem.-II

Matrix Algebra & Graph Theory

Time : 2:30 Hours] **[Max. Marks : 70**

Instruction : Use of simple calculator is allowed.

1. (a) Attempt any **one**. **10**

(i) Suppose G is a graph with at least two vertices. Show that it is impossible that all vertices have different degrees.

(ii) Let G be a k-regular graph, where k is an odd number. Prove that the number of edges in G is a multiple of k.

(iii) Prove that it is impossible to have a group of nine people at a party such that each one knows exactly five of the others in a party.

(b) Attempt **all** : **4**

(i) Define simple graph.

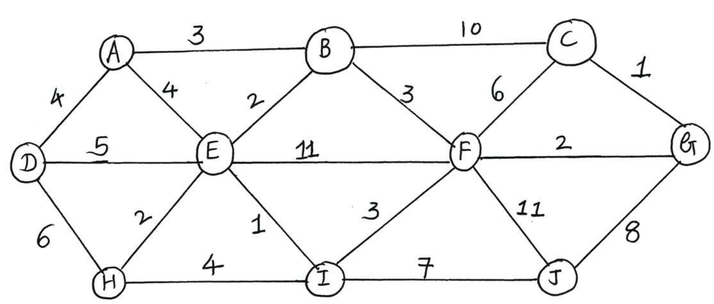
(ii) Give an example of 4-regular graph on 6 vertices. (Draw it).

(iii) Determine the number of edges in a graph having six vertices, two having degree 4 and four having degree 2.

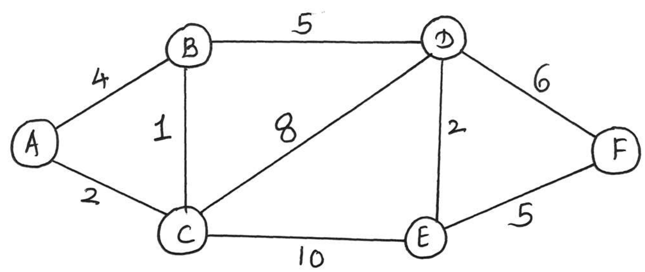
(iv) Define square of a graph.

2. (a) Attempt any **one**. **10**

(i) Find minimal spanning tree of the following graph using Kruskal's algorithm.



(ii) Apply Dijkstra's algorithm to find shortest path between a vertex A and F.



(b) Attempt **all** : **4**

(i) Define tree

(ii) Define digraph.

(iii) Define weakly connectedness in a digraph.

(iv) Define ditrail.

3. (a) Attempt any **one**. **10**

(i) Find the inverse of a matrix $A = \begin{bmatrix} -1 & 2 & 3 \\ 2 & -1 & 3 \\ 3 & -1 & 2 \end{bmatrix}$ and hence verify $AA^{-1} = I_{3 \times 3}$.

(ii) Prove that K_5 is not a planar graph.

(b) Attempt **all** : **4**

(i) If $A = \begin{bmatrix} -1 & 3 \\ 4 & 2 \\ 2 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 2 & -2 \\ -1 & 3 & 1 \end{bmatrix}$. Find AB and BA.

(ii) If A and B is $n \times n$ matrices then show that trace $(A + B)$ = trace A + trace B.

If A and B is $n \times n$ matrices then show that trace $(\alpha A) = \alpha$ trace A .

4. (a) Attempt any **one**. **10**

(i) Verify Caley-Hamilton's theorem for a matrix $A = \begin{pmatrix} 2 & 0 & 1 \\ -2 & 3 & 4 \\ -5 & 5 & 6 \end{pmatrix}$

(ii) Find row reduced echelon form of a matrix $A = \begin{pmatrix} -2 & -1 & 3 & -3 \\ 3 & 2 & -1 & 5 \\ -2 & -1 & 2 & 0 \end{pmatrix}$

(b) Attempt any **one**. **4**

(i) Find the rank of a matrix $A = \begin{pmatrix} 2 & 1 & 2 \\ 1 & 2 & 2 \\ 2 & 1 & 2 \end{pmatrix}$.

(ii) Show that the subset A = {(5, 0, 0), (0, 1, 2), (0, 2, 7)} of the vector space $\mathbb{R}^3$ is linearly independent.

5. (a) Attempt any **one**. **10**

(i) Three consecutive coefficients in the expansion of $(1 + x)^n$ are 28, 56 and 70. Find n.

(ii) How many permutations are possible with all the letters of the word HEXAGON ? In the dictionary order of these words, which place will this word occupy ?

(b) Attempt any **two** : **4**

(i) If repetition is allowed, how many 3×3 matrices can be formed using numbers 0, 1, 2 ?

(ii) Find the constant term in the expansion of $\left(\sqrt{\frac{x}{3}} + \frac{3}{2x^2}\right)^{10}$

(iii) Find the co-efficient of x^{32} in the expansion of $\left(x^4 - \frac{1}{x^3}\right)^{15}$