Seat No. : _______________

MN-140

March-2019

B.Sc., Sem.-VI

CC-308 : Mathematics
(Analysis-II)

Time : 2:30 Hours] **[Max. Marks : 70**

Instructions : (1) All the **four** questions are compulsory.

(2) Each of the questions **Q.1** and **Q.2** are of **18** marks and **Q.3, Q.4** are of **17** marks.

1\. (A) Write the following :

(i) Define Riemann integrable function on [a, b] . Prove : If f is a monotone function on [a, b] then f is Riemann integrable on [a, b]. Show that fg $\in$ R [a, b] whenever f, g $\in$ R [a, b]. **7**

(ii) If $P = \left\{0, \frac{1}{n}, \frac{2}{n}, \frac{3}{n}, \ldots, \frac{n}{n}\right\}$ is a partition of [0, 1] and $f(x) = \frac{x}{2}$ then find $\lim_{n\to\infty} U_p(f)$ $\lim_{n\to\infty} L_p(f)$. **7**

OR

(i) State and prove First Mean Value Theorem of Integral Calculus. Verify it for the function $f(x) = x + 1$ on [0, 1].

(ii) Prove that $\frac{\pi^3}{3b} \le \int_0^{\pi} \frac{x^2 dx}{a\cos^2 x + b\sin^2 x} \le \frac{\pi^3}{3a}$ where, $0 < a < b$.

(B) Attempt any **two** out of **three** in short : **4**

(i) State the fundamental Theorem of Calculus.

(ii) Evaluate : $\int_{-2}^{2} x|x|\, dx$.

(iii) Show that the constant function is Riemann integrable.

2. (A) Write the following :

(i) Define conditional convergence of the series.

If (a_n) is a decreasing sequence of positive terms converging to zero, then prove that the series $\sum_{n=1}^{\infty} (-1)^{n+1} a_n$ converges. Discuss the convergence of the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n}$. **7**

(ii) If $\sum_{n=1}^{\infty} a_n$ is a convergent series of real numbers then show that

$\sum_{n=1}^{\infty} \frac{\sqrt{a_n}}{n}$ is also convergent. Discuss the absolute convergence of the series

$\sum_{n=1}^{\infty} (-1)^{n+1} \left(\frac{\sqrt{n^2+2}-n}{\sqrt{n}} \right)$. **7**

OR

(i) State and prove Cauchy's condensation test and hence establish p-test $\Sigma \frac{1}{n^p}$ for the series.

(ii) If $\sum_{n=1}^{\infty} a_n$ diverges and all $a_n > 0$ then show that the series $\Sigma \frac{a_n}{1+na_n}$ is divergent.

(B) Attempt any **two** out of **three** in short : **4**

(i) Discuss the convergence of the series $\sum_{n=2}^{\infty} \frac{1}{n \log n}$

(ii) The series $\sum_{n=0}^{\infty} \frac{1}{2^{2n}}$ converges to the sum _______.

(iii) If (a_n) is a decreasing sequence of positive terms and Σa_n is convergent then $\lim_{n \to 0} na_n = 0$. Give suitable name to this statement.

3. (A) Write the following :

(i) If the series Σa_n is absolutely convergent then prove that any rearrangement of Σa_n has the same sum. **7**

(ii) Discuss the convergence of the following power series stating interval of convergence : **7**

(1) $\sum_{n=1}^{\infty} \frac{x^n}{4^n n^2}$

(2) $\sum_{n=1}^{\infty} \frac{n^n x^n}{n!}$

(3) $\sum_{n=0}^{\infty} \frac{2^n x^n}{3^n + 4}$

OR

(i) Prove : If f is defined on $[a, \infty)$ and $\int_a^{\infty} |f|$ converges then $\int_a^{\infty} f$ converges,

and $\left| \int_a^{\infty} f \right| \leq \int_a^{\infty} |f|$.

(ii) Examine the convergence of the following improper integrals :

(a) $\int_2^{\infty} \frac{1}{x(x-1)} dx$

(b) $\int_1^{\infty} \frac{\cos x}{x^3} dx$

(c) $\int_{-\infty}^{\infty} \frac{1}{1+x^2} dx$

(B) Attempt any **two** out of **three** in short : **3**

(i) Find the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{n^3 x^n}{4^n}$.

(ii) Find the Cauchy product of $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ with itself.

(iii) Evaluate : $\int_{-2}^{2} x|x|dx$

4. (A) Write the following :

(i) State Taylor's theorem. Using Lagrangian form of the remainder show that

$\log(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$. For $-1 < x \leq 1$ **7**

(ii) State and prove Binomial series theorem. **7**

OR

(i) Show that $(1 + x)^a \approx 1 + \sum_{n=1}^{\infty} \frac{\alpha(\alpha-1)(\alpha-2) \ldots\ldots (\alpha-n+1)}{n!} x^n$ converges and find its radius of convergence. Give suitable name to this result.

(ii) Obtain in power series solution of the differential equation $(1 - x)\, y' + 2y = 0$ with the initial condition $y(0) = 2$.

(B) Attempt any **two** out of **three** in short : **3**

(i) Obtain the Maclaurin series expansion for $\log(1 + x)$.

(ii) If $y(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots\ldots$ and $y(x) = y'(x)$ then obtain the relation between the coefficients.

(iii) Examine the validity of the statement $\ln 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots.$
