Seat No. : _______________

MP-128

March-2019

B.Sc., Sem.-VI

310 : Mathematics

Time : 2:30 Hours] **[Max. Marks : 70**

1. (A) (1) Define the following term with graph : **7**

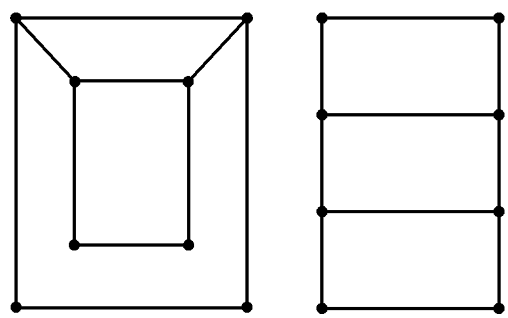
(i) Adjacent vertices

(ii) Null graph

(iii) Edge deleted sub graph

(iv) k-regular graph

(2) Define isomorphism of a graph. Discuss whether the following graphs are isomorphic or not ? **7**



OR

(1) For any graph G with e edges and n vertices $v_1, v_2, v_3, \ldots \ldots \ldots v_n$ Prove that

$$\sum_{i=1}^{n} d(v_i) = 2e.$$ **7**

(2) Define k-cube Q_K and prove that Q_K has 2^K vertices and 2^{k-1} edges. **7**

(B) Answer in short : (Any **TWO**) **4**

(i) What is the smallest positive integer n such that complete graph Kn has at least 600 edges.

(ii) Draw 3 regular graph with 5 vertices.

(iii) Define neighbourhood set with example.

2. (A) (1) Let G be acyclic graph with n vertices and k connected components, then prove that G has a n-k edges. **7**

(2) Without drawing actual graph, determine whether the graph is connected or not, whose adjacency matrix is $A(G) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 2 & 2 \\ 0 & 2 & 1 & 2 \\ 0 & 2 & 2 & 1 \end{bmatrix}$. **7**

OR

(1) If T is a tree with n vertices then prove that it has precisely n-1 edges. **7**

(2) (i) Let u and v be distinct vertices of a tree T. Then there is precisely one path from u to v.

(ii) Let G be graph without any loops. If for every pair of distinct vertices u and v of G there is precisely one path from u to v, then G is a tree. **7**

(B) Answer in short : (Any **TWO**) **4**

(i) Define forest with graph.

(ii) Draw star graph $K_{1,6}$

(iii) Give two trees with Five vertices.

3. (A) (1) State and prove Cayley theorem. **7**

(2) Let G be a simple graph on n vertices. G has K components then the number of edges of G satisfies $n - k \leq m \leq \frac{(n-k)(n-k+1)}{2}$. **6**

OR

(1) Let G be a graph with n vertices, where $n \geq 2$. Then G has atleast two vertices which are not cut vertex. **7**

(2) Prove that any simple graph with n vertices and more than $\frac{(n-1)(n-2)}{2}$. **6**

(B) Answer in short : (Any **TWO**) **4**

(i) Draw Petersen graph.

(ii) Define Cut – Vertex with example.

(iii) Let G be a connected graph with 14 edges then what is the maximum possible number of vertices in G ?

4. (A) (1) A connected graph G is Euler if and only if the degree of every vertex is even. **7**

(2) Prove that the vertex connectivity k(G) of graph G is always less than or equal to the Edge connectivity $\lambda(G)$. **6**

OR

(1) If G is a simple graph with $n \geq 3$ vertices and if deg V + degW $\geq$ n for each pair of non-adjacent vertices V and W then G is Hamiltonian. **7**

(2) Discuss The Konigsberg bridges problem. **6**

(B) Answer in short : (Any **TWO**) **4**

(i) How many different Hamiltonian cycle for complete graph k_5.

(ii) Define Hamiltonian path and Hamiltonian cycle.

(iii) Define Closure of graph G.