Seat No. : _______________

MD-131

March-2019

B.Sc., Sem.-V

CC-303 : Mathematics
(Complex Variables & Fourier Series)

Time : 2:30 Hours] **[Max. Marks : 70**

Instructions : (1) **All** the questions are compulsory.
(2) Question **1** and **2** are of **18** marks.
(3) Question **3** and **4** are of **17** marks.

1. (A) (1) State and prove de Moiver's theorem. **7**
(2) Prove that $\cos(z_1 + z_2) = \cos z_1 \cos z_2 - \sin z_1 \sin z_2$. **7**

OR

(1) Define sine hyperbolic and cosine hyperbolic functions. Also prove that $\sin(iy) = i \sinh y$ and $\cos(iy) = \cosh y$. **7**
(2) For complex numbers z_1 and z_2 prove that $||z_1| - |z_2|| \le |z_1 \pm z_2| \le |z_1| + |z_2|$. **7**

(B) Answer any **two** in short : **4**
(1) Define sin z and cos z functions.
(2) Define convergence of a sequence.
(3) Prove $\sin(-z) = -\sin z$.

2. (A) (1) Derive Cauchy-Riemann equations in Cartesian form .i.e. $u_x = v_y$ and $u_y = -v_x$. **7**
(2) If $u = x^2 - y^2 - 2xy - 2x + 3y$ then find harmonic conjugate v, also find $f(z) = u+iv$ in the form of z. **7**

OR

(1) Derive Cauchy-Riemann equations in polar form. **7**
(2) Prove that $f(z) = |z|^2$ is continuous everywhere but nowhere differential except at the origin. **7**

(B) Answer any **two** in short : **4**
(1) Define limit of function at a point.
(2) Define continuity of function at a point.
(3) Define Harmonic function..

3. (A) (1) Prove that an analytic function f(z) is conformal at z_0 iff $f'(z_0) \neq 0$. **7**

(2) Find implicit form which maps $z_1 = 1$, $z_2 = 0$ and $z_3 = -1$ onto $w_1 = i$, $w_2 = \infty$ and $w_3 = 1$. **6**

OR

(1) Consider the map $w = ze^{i\pi/4}$ determine the region R' of w-plane corresponding to the triangular region bounded by the lines $x = 0$, $y = 0$, $x + y = 1$ in z-plane. **7**

(2) Find Mobius transformation which maps $z_1 = -1$, $z_2 = 0$ and $z_3 = 1$ onto $w_2 = -i$, $w_2 = 1$, and $w_3 = i$. **6**

(B) Answer any **two** in short : **4**

(1) Define implicit form.

(2) Define Mobius transformation.

(3) Define conformal mapping.

4. (A) (1) State and prove Bessel's inequality. **7**

(2) Obtain Fourier series expansion of $f(x) = x \sin x$. Hence deduce that **6**

$$\frac{\pi}{4} = \frac{1}{2} + \frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} \ldots \ldots \ldots .$$

OR

(1) Find Fourier series for the function $f(x) = x^2$ in $[-\pi, \pi]$ and deduce that

(i) $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \ldots \ldots \ldots = \frac{\pi^2}{6}$

(ii) $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots \ldots \ldots = \frac{\pi^2}{12}$

(iii) $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \ldots \ldots \ldots = \frac{\pi^2}{8}$

(2) Find Fourier series for the function $f(x) = x$ in $[-\pi, \pi]$.

(B) Answer any **two** in short : **4**

(1) Define fourier series.

(2) Prove that $\int_{-\pi}^{\pi} \cos nx \, dx = 0$, for all n.

(3) Define triangular series.
