Seat No. : _______________

MF-122

March-2019

B.Sc., Sem.-V

305 : Mathematics

(Discrete Mathematics)

Time : 2:30 Hours] **[Max. Marks : 70**

Instructions : (1) **All** the questions are compulsory.
(2) Notations and Terminology are standard.
(3) Figures to the right indicates the full marks.

1. (a) (1) Prove that every chain is Lattice. Does converse of this theorem is true or not ? Justify your answer. **9**
(2) Explain Hass Diagram and also draw the Hass Diagram of (S_{210}, D). **9**

OR

(1) State distributive Inequality and prove any one of them. **9**
(2) For a Lattice $(L, \leq)$, prove that $a \leq b \Leftrightarrow a * b = a \Leftrightarrow a \oplus b = b, \forall a, b \in L$. **9**

(b) Short Answer : (Any **three** out of **five**) **6**
(1) Find LUB of 10 and 6 in the lattice $\langle S_{30}, D\rangle$.
(2) Define : Partially ordered relation.
(3) Define : Lattice.
(4) Give a relation on the set which is Transitive but neither reflexive nor symmetric.
(5) Give an example of a Poset which is not a Latice.

2. (a) (1) State De'Morgan's law and prove any one of them. **9**
(2) Show that $(S_{30}, *, \oplus)$ and $(P(A), \cap, \cup)$ are isomorphic Lattice for $A = \{a, b, c\}$. **9**

OR

(1) Prove that in Distributive lattice if an element has a complement then it must be unique. **9**
(2) For a complemented distributive Lattice $(L, *, \oplus, 0, 1)$, prove that $a \leq b \Leftrightarrow a * b' = 0 \Leftrightarrow b' \leq a' \Leftrightarrow a' \oplus b' = 1, \forall a, b \in L$ **9**

(B) Short Answer : (Any **three** out of **five**) **6**

(1) Define : Sub lattice with an example.

(2) Define : Bounded Lattice.

(3) Define : Lattice Homomorphism.

(4) Give an example of a lattice which is complemented but not distributive.

(5) Draw $L_1 \times L_2$ where L_1 and L_2 are divisors of 4 and 9 respectively.

3. (A) (1) Prove that the only elements which are atoms in a Boolean algebra are those which covers zero. **8**

(2) Obtain the SOP and POS canonical forms of the Boolean expression $a(x_1, x_2, x_3) = (x_1 * x_2') \oplus x_3$. **8**

OR

(1) State and prove stone representation theorem.

(2) Define equivalent Boolean expression also show that $(x \oplus y) * (x' \oplus z)$ and $(x * z) \oplus (x' * y)$ are equivalent.

(B) Short Answer : (Any **three** out of **five**) **6**

(1) Define an Atom.

(2) Define Minterm.

(3) Find A(12) and A(60) for (S_{120}, D).

(4) Define Symmetric Boolean expression.

(5) Define Boolean algebra.

Seat No. : _______________

MF-122

March-2019

B.Sc., Sem.-V

305 : Mathematics
(Number Theory)

Time : 2:30 Hours] **[Max. Marks : 70**

1. (A) (1) State and prove Division algorithm theorem. **9**

(2) Using the Euclidean algorithm to obtain the integer x and y such that gcd(12378, 3054) = 12378x + 3054y. **9**

OR

(1) Define Linear Diophantine equation. Find the solution of linear Diophantine Equation 54x + 21y = 906 in positive integers. **9**

(2) Prove that there are infinite number of primes of the form 4n + 3. **9**

(B) Attempt any **three** (in short) : **6**

(1) If p is a prime number and p/ab, then prove that p/a or p/b.

(2) Is Diophantine equation 2x + 6y = 9 has solution ? Justify your answer.

(3) A number 360 can be written as product of prime in canonical form.

(4) Find the L.C.M for two integers 306 and 657.

(5) Define prime and relatively prime.

2. (A) (1) In usual notation prove that $2^{20} \equiv 1 \pmod{41}$ and find the remainder when the sum l! + 2! + 3! +... + 100! is divisible by 12. **9**

(2) Does there exists a solution of the congruence $15x \equiv 9 \pmod{12}$? If so, find out all mutually congruent solution of it. **9**

OR

(1) Define congruence relations and prove that it is an equivalence relation. **9**

(2) Using Chinese remainder theorem, find integer x such that $2x \equiv 1 \pmod 3$ $3x \equiv 1 \pmod 5$; $5x \equiv 1 \pmod 7$. **9**

(B) Attempt any **three** (in short) : **6**

(1) Solve : $18x \equiv 30 \pmod{42}$.

(2) Define complete residue system modulo.

(3) If g.c.d(a, b) = d then prove that g.c.d $\left(\frac{a}{d}, \frac{b}{d}\right) = 1$.

(4) Prove that $n^4 + 4$ is composite number for $n \in N$ with $n > 1$.

(5) Prove that the number N = 1571724 is divisible by 9 and 11.

3. (A) (1) State and prove Wilson's theorem. **8**

(2) If p and q are distinct primes such that $a^p \equiv a \pmod{q}$ and $a^q \equiv a \pmod{p}$ then prove that $a^{pq} \equiv a \pmod{pq}$. **8**

OR

(1) State and prove the Euler's theorem. **8**

(2) In usual notation show that $(1835^{1910} + 1986^{2061}) \equiv 0 \pmod{7}$. **8**

(B) Attempt any **three** (in short) : **6**

(1) Define Euler's Phi-function.

(2) Prove that n is prime if and only if $\phi(n) = n - 1$.

(3) Find ϕ (300).

(4) What is the formula for ϕ (p^k) and ϕ (pq) where p and q are distinct prime.

(5) State Fermat's theorem.