Seat No. : _______________

MC-118

March-2019

B.Sc., Sem.-V

302 : Mathematics
(Analysis-I)

Time : 2:30 Hours] **[Max. Marks : 70**

Instructions : (1) **All** the questions are compulsory.
(2) Notations are usual, everywhere.
(3) Figures to the right indicate marks of the question/sub-question.

1. (A) (i) State and prove Archimedean Property. Using that prove that if $S = \{ 1/n : n \in N\}$, then $\inf S = 0$. **7**

(ii) Prove that the set $N \times N$ is denumerable. **7**

OR

(i) State and prove (rational) density theorem.

(ii) Prove that $\sqrt{11}$ is ir-rational.

(B) Attempt any **two** in short : **4**

(1) Define countable set. Give an example of countable proper subset of N.

(2) Determine the set A of all real number x such that $2x + 3 \leq 6$.

(3) Give examples of two disjoint uncountable proper subsets of R.

2. (A) (i) Prove that a sequence of real numbers is Cauchy iff it is convergent. **7**

(ii) Using definition show that the sequence $\left\{\frac{n^2 + 1}{n + 100}\right\}$ diverges to ∞. **7**

OR

(i) State and prove Bolzano-Weierstrass theorem.

(ii) If $S_n = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + ... + \frac{1}{n!}$ then prove that $2 < \lim\limits_{n \to \infty} S_n < 3$.

(B) Attempt any **two** in short : **4**

(1) Define convergent sequence. Write the limit of sequence $1, 0, \frac{1}{2}, 0, \frac{1}{3}, 0, \frac{1}{4}, 0, \frac{1}{5}, ...$ if it is convergent.

(2) Give an example of a divergent sequence $\{x_n\}$ but $\{x_n^2\}$ converges to 2.

(3) Every monotonic sequence is convergent. True or false ?

3. (A) (i) State and prove intermediate value theorem. **7**

(ii) Using definition verify $\lim_{x \to 6} x^2 + 2x - 7 = 41$, also find δ corresponding for $\varepsilon = 1.1$ **6**

OR

(i) If function f is continuous at point a and $\{x_n\}$ is a sequence converging to a, then prove that the sequence $\{f(x_n)\}$ is convergent to $f(a)$.

(ii) Define uniform continuity of function. Discuss the uniform continuity of the functions $f(x) = \frac{1}{x}$ on $[0, \infty)$.

(B) Attempt any **two** in short : **4**

(1) Define continuity of function.

(2) Give an example of real function which is discontinuous only at two points.

(3) Define uniform convergence of sequence.

4. (A) (i) Suppose the function fog is defined in a neighborhood of point x_0, and that g is differentiable at x_0, and f is differentiable at point $y_0 = f(x_0)$ then prove that fog is differentiable at point x_0. **7**

(ii) State mean value theorem and verify it for $f(x) = x^3 - 3x + 2$ in $[-1, 2]$, find appropriate c. **6**

OR

(i) State and prove Darboux's theorem.

(ii) Evaluate : $\lim_{x \to 0} \left(\frac{2^x + 3^x + 5^x}{3}\right)^{\frac{1}{x}}$, & $\lim_{x \to 0} \frac{e^x - 2 - x - (x^2/3)}{\sin^3 x}$

(B) Attempt any **two** in short : **4**

(1) State the Chain's rule for differentiation.

(2) State roll's theorem.

(3) State only L'Hospital rule 1^{st}.