Seat No. : _______________

AB-165

April-2019

B.Sc., Sem.-IV

CC-204 : Statistics

Random Variable and Probability Distribution-II

(Old)

Time : 2:30 Hours] **[Max. Marks : 70**

Instructions : (1) **All** questions are compulsory.
(2) Each question carries equal and **14** marks.
(3) Statistical tables will be provided on request. Use of scientific calculator is allowed.

1. (a) (i) Stating the purpose of Bonferronie inequality, prove it. **7**
(ii) In usual notations, derive relation between characteristic function and raw moments. **7**

OR

(i) Define Characteristic function and state its properties. **7**
(ii) In usual notations, prove $P[g(x) > t] \leq \frac{E(g(x))}{t}$, $t > 0$ **7**

(b) Answer any **two** of the following : **4**
(i) State Inversion Theorem on characteristic function.
(ii) State Boole's inequality and its use.
(iii) State measure of dispersion, on which Chebyshev's inequality is based. State its importance.

2. (a) (i) In usual notations, $X \sim N(\mu, 16)$, show that median and mode of normal distribution are same and equal to μ. **7**
(ii) If X and Y are two independent Gamma variates with parameters (α, β) and (α, λ) respectively, then derive the distribution of X / Y. **7**

OR

(i) In usual notations, for Weibull distribution with two parameters (α, β), derive expressions for quartile deviation. **7**
(ii) State and prove additive property of Normal distribution. **7**

(b) Attempt any **two** for the following : **4**
(i) If X and Y are two independent Gamma variates with parameters (α, β) and (α, λ) respectively, then state the distribution of X + Y. Also, state the mode of gamma variate with parameters (α, β).
(ii) State the moment generating function of Normal distribution. Hence, obtain first two cumulants.
(iii) State two characteristics of normal distribution.

3. (a) (i) Define Two dimensional random variables, joint probability mass function, Discrete joint probability distribution function. **7**

(ii) In usual notations, prove that $E(X) = E(E(Y/X))$. **7**

OR

(i) If the joint probability distribution of random variables (X, Y) be : **7**

Y	X		
	−1	0	1
2	1/8	2/8	1/8
3	1/8	1/8	2/8

Then, find (1) $P(X = 1, Y = 2)$, $P(X = 0, Y > 1)$, (2) marginal distribution of X and (3) conditional distribution of Y given X.

(ii) Define conditional expectation. Hence or otherwise, in usual notations, prove that $V(X) = E(V(X)) + V(E(X))$. **7**

(b) Attempt any **three** of the following : **3**

(i) Define the independence of random variables.

(ii) State the value of $E(E(Y/X))$.

(iii) Give one example which uses two dimensional random variables.

(iv) Define product moment.

(v) Define Karl Pearson's correlation co-efficient.

4. (a) (i) In usual notations, State and prove Chapman Kolmogorov equation. **7**

(ii) For the Markov chain $\{X_n, n = 1, 2, ...\}$, with the state space $S = \{1,2,3\}$ and the transition matrix P as shown below : **7**

$$\begin{pmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{pmatrix}$$

and the initial distribution is $\Pi_0 = (0.7, 0.2, 0.1)$, then obtain following :

$P\{X_2 = 3\}$, $P\{X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2\}$

OR

(i) With respect to Markov chain, define terms giving one illustration for each : irreducible (or ergodic) Markov chain, time homogeneous Markov chain, one step transition probability, communicative state. **7**

(ii) What is Markov chain ? With respect to Markov chain, define transition matrix. Also, state properties of transition matrix. **7**

(b) Answer any **three** of the following : **3**

(i) Define Accessible states.

(ii) Define persistent (recurrent) states.

(iii) What is transitive state ?

(iv) Define with respect to Markov chain state space.

(v) Give one practical situation, where Markov chain is applied.

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AB-165

April-2019

B.Sc., Sem.-IV

CC-204 : Statistics
Distribution Theory-II
(New)

Time : 2:30 Hours] **[Max. Marks : 70**

Instructions :
(1) Figures to the right indicate full marks of the question/sub-question.
(2) Notations used in this question paper carry their usual meaning.
(3) Use of scientific calculator is allowed.
(4) Statistical & logarithmic tables and graph papers will be provided on request.

1. (a) (i) State probability mass function of negative binomial distribution and obtain its moment generating function. **7**
(ii) Define Hypergeometric distribution. Also, derive mean and variance of Hypergeometric distribution. **7**

OR

(i) If a random variable X follows geometric distribution, then in usual notations, derive mean and variance of X. **7**
(ii) In usual notations, obtain recurrent relation for the central moments of negative binomial distribution. **7**

(b) Attempt any **two** of the following : **4**
(i) State applications of Hypergeometric distribution.
(ii) State the relation between mean and variance of negative binomial distribution.
(iii) State memoryless property of geometric distribution.

2. (a) (i) Define Weibull distribution. In usual notations, obtain mean and first quartile of two parameter Weibull distribution. **7**
(ii) Define lognormal distribution. Derive an expression mean and variance of Lognormal distribution. **7**

OR

(i) Derive mean and variance of Laplace distribution. **7**
(ii) Derive characteristic function of Cauchy distribution. **7**

(b) Attempt any **two** of the following : **4**
(i) Give applications of Weibull distribution.
(ii) State probability distribution function of Cauchy distribution. For Cauchy distribution, mean does not exist. Give reason.
(iii) State difference between Double Exponential and Laplace distributions.

3. (a) (i) State probability density function of Normal distribution with parameters μ and σ^2. Derive median and mode of N (μ, σ^2). **7**

(ii) Derive expression for central moments of Normal distribution with parameters μ and σ^2. **7**

OR

(i) With respect to Bivariate Normal distribution, define marginal and conditional distributions. **7**

If X and Y have bivariate normal distribution with parameters (3,1, 16, 25, 0.6), Determine the following probabilities (A) Pr $[3 < Y < 8]$, (B) Pr $[3 < Y < 8/X = 7]$

(ii) State and prove two random variable (X, Y) following Bivariate Normal Distribution, are independent if and only if $\rho = 0$. **7**

(b) Attempt any **three** of the following : **3**

(i) The curve of probability density function of $N(\mu, \sigma^2)$ is symmetric. Do you agree ?

(ii) Define independence of random variables (X,Y), if they follow Bivariate Normal distribution.

(iii) State moment generating function of $N(\mu, \sigma^2\}$.

(iv) State mean of the conditional distribution of Y given X, where (X,Y) follow Bivariate Normal distribution.

(v) If (X,Y) follow Bivariate Normal distribution, and if $\rho = 0$, then state the probability density functions of X and Y.

4. (a) (i) State and prove Bernoulli's Law of large numbers. **7**

(ii) Examine whether the weak law of large numbers holds good for the sequence

X_n, n = l, 2, 3, ... of n independent variables where $P\left(X_n = \frac{1}{\sqrt{n}}\right) = \frac{2}{3}$, and

$P\left(X_n = \frac{1}{\sqrt{n}}\right) = \frac{1}{3}$. **7**

OR

(i) In usual notations, state and prove weak law of large number. **7**

(ii) Define characteristic function. State its properties. Also, derive relation between raw moments and characteristic function. **7**

(b) Attempt any **three** : **3**

(i) Define convergence in probability.

(ii) State Lindberg Levi's and Liapounoff's form of Central Limit Theorem.

(iii) State inversion theorem on characteristic function.

(iv) Give one application of Central Limit Theorem.

(v) State one use of Weak Law of Large Numbers.