Seat No. : _______________

JE-101

January-2021

B.Sc., Sem.-III

201 : Statistics

(Distribution Theory – I)

(New Course)

Time : 2 Hours] **[Max. Marks : 50**

Instructions : (1) There are **two** sections in this question paper.
(2) **All** questions in Section – I carry equal marks.
(3) Attempt any **three** questions from Section – I.
(4) Section – II is compulsory.
(5) Figures to the right indicate full marks of the questions/sub-questions.

1. (a) State probability mass function of Poisson distribution and in usual notations; derive additive property of Poisson distribution. **7**

   (b) If a random variable X ~ Bn(n, p), in usual notations; show that cumulant generating function of X is $M(t) = (q + pe^t)^n$. Also, obtain first two central moments. **7**

2. (a) What is Truncation ? **7**
   Derive truncated binomial distribution. Also, obtain its variance.

   (b) In usual notations, derive recurrent relation for central moments of Poisson distribution with parameter m. **7**

3. (a) Identify the probability distribution a random variable X if its form is **7**

$$F(x) = \begin{cases} 0 \ , & x \le 0 \\ x & 0 < x < 1 \\ 1 \ , & x \ge 1 \end{cases}$$

   Hence or otherwise, obtain mean and variance of X.

   (b) State an application of exponential distribution. **7**
   A random variable X an exponential distribution with parameter α, in usual notations, derive the first two raw moments X.

4. (a) For beta type I distribution, derive its mean and harmonic mean. **7**

   (b) Random variables X and Y are independent gamma variates with parameters (α, β) and (α, λ) respectively, then show that Z = X + Y follows gamma distribution. Also, find $P[0 < z < 2]$ when $\beta = 1/2$ and $\lambda = 1/2$. **7**

5. (a) What is Jacobian of transformation ? State its uses in probability distribution theory. **7**

(b) If X and Y are independent random variables, in usual notations, derive the probability density function of $U = X + Y$. **7**

6. (a) If the cumulative distribution of X is F(X), then obtain the cumulative distribution function and probability function of (i) $Y = X + 3$, (ii) $Y = 2X^2$ **7**

(b) Let X be a continuous random variable with probability density function $f(x)$, If $Y = g(x)$ is monotonically increasing or decreasing function of X, then the probability density function of Y is $h(y) = f(x) \left| \frac{dx}{dy} \right|$ **7**

7. (a) Define order statistics. **7**

State joint probability density function of the largest order statistics.

(b) For a rectangular distribution with the probability density function of random variable X is $f(x) = \begin{cases} \frac{1}{b-a} , & a < x < b \\ 0 , & \text{otherwise} \end{cases}$

If a random sample (X_1, X_2, X_3) of size 3 is taken on X, derive probability distribution of the smallest order statistics (Y_1). Also, find $P[Y_1 < 0.3]$. **7**

8. (a) Derive probability distribution of sample range of order statistics. **7**

(b) In usual notations, derive the probability density function of the smallest order statistics (Y_1)**.** **7**

Section – II

9. Attempt any **eight**. **8**

(i) State one use of order statistics.

(ii) State mean and variance of Bernoulli distribution.

(iii) State the distribution of sum of n independent Bernoulli variates.

(iv) Give two applications of Poisson distribution.

(v) Write the value of first two cumulants of Poisson distribution.

(vi) State probability density function of beta type II distribution. Also, state its mean value.

(vii) If $X \sim G(a, m)$ and $Y \sim G(a, n)$ be two independently distributed gamma variates, then state the distribution of X/Y.

(viii) State skewness of a random variable $X \sim G(\alpha, \beta)$.

(ix) State probability density function of the smallest order statistics (Y_1).

(x) State joint probability density function of $U = U(x, y)$, and $V = V(x, y)$, given the joint probability density function of (X, Y).
