Seat No. : _______________

JC-126

July-2021

B.Sc., Sem.-VI

308 : Mathematics
(Analysis - II)

Time : 2 Hours] **[Max. Marks : 50**

Instructions : (1) Attempt any **three** questions in Section – **I**.
(2) Section – **II** is a *compulsory* section of short questions.
(3) Notations are usual everywhere.
(4) The right hand side figures indicate marks of the sub-questions.

SECTION – I

1. (A) If $f \in R[a, b]$ and $g \in R[a, b]$, then prove that $f + g \in R[a, b]$ and

$$\int_a^b (f+g) = \int_a^b f + \int_a^b g.$$ **7**

(B) Let $f(x) = 2x$ on $[0, 1]$. For $n \in \mathbb{N}$, define

$P_n = \left\{0, \frac{1}{n}, \frac{2}{n}, \frac{3}{n}, \ldots, \frac{n-1}{n}, 1\right\}$ then compute $\lim\limits_{n \to \infty} U_{P_n}$ and $\lim\limits_{n \to \infty} L_{P_n}$. Is the function integrable ? If so, find the value of the integral. **7**

2. (A) State and prove First Fundamental Theorem of Calculus. **7**

(B) State Second Mean Value Theorem of Integral Calculus. Find a point c in $\left[0, \frac{\pi}{2}\right]$ such that $\int_0^1 \frac{1}{1+x^2}\,dx = 1.$ **7**

3. (A) Let $\{x_n\}$ and $\{y_n\}$ be real sequences. Then prove that

(i) $\inf x_n \le \underline{\lim}\, x_n \le \overline{\lim}\, x_n \le \sup x_n$

(ii) $\overline{\lim}\,(x_n + y_n) \le \overline{\lim}\, x_n + \overline{\lim}\, y_n$ **7**

(B) State and prove condensation test. Hence check the convergence of $\Sigma_{n=2}^{\infty} \dfrac{1}{n(\log n)^{\alpha}}, \alpha\in\mathbb{R}$. **7**

4. (A) Is every Cauchy sequence in $\mathbb{C}$ is convergent ? Justify. **7**

(B) State Cauchy's root test. Hence check the convergence of $\Sigma_{n=1}^{\infty} \dfrac{1}{(n+1)^{n}}$. **7**

5. (A) State and prove Fundamental Theorem on alternating series. **7**

(B) Define Conditionally convergent series. Check whether $\Sigma_{n=0}^{\infty} (-1)^{n} \dfrac{1}{\sqrt{n+1}}$ is conditionally convergent. **7**

6. (A) If $\Sigma |a_n|$ converges, then prove that the series Σa_n converges. Is the converse true ? Justify. **7**

(B) Find the radius of convergence of the following power series whose n^{th} terms are given below :

(1) $(2n+1)z^{n}$ (2) $\dfrac{n^2}{n!} z^{n}$ **7**

7. (A) State and prove Binomial series theorem. **7**

(B) Derive Taylor's formula with the integral form of the remainder for $f(x) = \cos x$ about $a = 0$ in $(-\infty, \infty)$. **7**

8. (A) For $-1 < x < 1$, prove that $\log(1+x) = x - \dfrac{x^2}{2} + \dfrac{x^3}{3} - \cdots + (-1)^{n-1}\dfrac{x^n}{n} + \cdots$ (Use Cauchy's form of remainder). **7**

(B) Find a power series solution of $y'' + 4y = 0$ with $y(0) = 1$ and $y'(0) = 0$. **7**

SECTION – II

9. Attempt any **four** short questions : **8**

(i) Does $|f| \in R[a, b]$ implies $f \in R[a, b]$? Justify.

(ii) Discuss convergence of the series $\Sigma_{n=1}^{\infty} \dfrac{1}{2^n}$.

(iii) Give example of a convergent series which is not absolutely convergent.

(iv) The Taylor's series of f converges to $f(x)$ at $x = a$ iff $R_n(x) \to$ _______ as $n \to$ _______. (Fill in the blanks)

(v) If $\lim_{n\to\infty} x_n \neq 0$ then $\Sigma_{n=0}^{\infty} x_n$ is _______ .

(vi) Write series of e^x. Is e rational ?